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Managing quality and pricing during a product recall: An analysis of pre-crisis, crisis and post-crisis regimes

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Abstract : Product recalls are often consequences of quality failures. While such failures are related to a manufacturer's or supplier's design quality, the perceived quality of products may be severely damaged when a product harm crisis occurs. However, most often, such a crisis will not last forever, and a firm at fault eventually recovers. Considering an optimal control model, we investigate the optimal pricing decisions, advertising and quality efforts of a firm while it anticipates a product recall and a subsequent recovery. We show that the decisions and profits of the manufacturer vary widely with the stochastic parameters: crisis likelihood, recovery likelihood, crisis impact and recovery intensity. We illustrate that myopic firms are more severely affected by a product recall than farsighted firms when the impact of recall is high. However, it might not be so detrimental to take myopic decisions for low impact recalls. In the absence of recovery, a product recall can lead to bankruptcy. High initial perceived quality may not insulate a firm against bankruptcy.

Keywords: Optimal control theory, product recall, advertising, pricing, quality

1 Introduction

Product recalls have been ravaging firms all over the world, with an increase in their number over the last decades, which is attributed to the complexity and globalization of contemporary supply chains. Product recalls may be minor when not much harm is caused and products are safely removed from the market. Such events receive low publicity and have little impact on the firm's reputation. Others, e.g., the recall of Boeing 737 Max, Takata airbags and Samsung note mobile phones, can cause immediate sizeable drop in sales and long-term goodwill and cause a serious financial impact for a firm (Cleeren et al., 2008; Min, 1989; Steven, 2015; Van Heerde et al., 2007), and, more importantly, consumer perception of product quality is severely affected after a product recall (Borah and Tellis, 2016; Tirunillai and Tellis, 2012).

A product recall can potentially lead to bankruptcy. For example, Westland/Hallmark Meat Packing Company recalled more than 143 million pounds of beef in 2008 as the meat was deemed unfit for consumption due to lack of quality inspection of the cattle. The company's recall costs initially exceeded \$116 million and in November 2012, Westland/Hallmark reached a \$500 million settlement with many plaintiffs, including the federal government. The California-based company became bankrupt (Canavan, 2013). Other brands have shown a capacity to rebound after a product recall crisis. To illustrate, the well-publicized recall of Samsung galaxy Note 7 due to exploding batteries lead to a drop in Samsung's premium cellphones' market share from 35% to 17% (Pressman, 2017). Samsung recalled all these phones with an estimated recall cost of \$5.3 billions (Lopez, 2017), and invested in quality. It regained consumer trust by advertising the eight point quality improvements it adopted (Fenech, 2017), and Samsung's sales boosted quickly within a few quarters. Another example is Toyota's notorious recall in February 2010 that caused a loss in product reliability ratings from 95% to 72%, while only 13% of the surveyed consumers thought that such a loss of reliability was permanent (Dong et al., 2021; Piotrowski and Guyette Jr, 2010). This attitude of consumers indirectly hints towards recovery of the firm in due time.

The consequences of a product recall can largely depend on how the firm reacts in terms of operations and marketing policies. In this article, we focus on how the perceived quality of a firm can deteriorate after a recall and how the firm can take quality, pricing and advertising decisions to minimize its impact. In our model, perceived quality is a multidimensional variable whose evolution depends on pricing, advertising and design quality decisions. Further, we assume that the manufacturer anticipates a product recall and an impending recovery, and it takes appropriate actions to mitigate the effect of product recall. This assumption is in line with both empirical and prescriptive research, which posits that firms learn to expect product recalls, and/or, make decisions while anticipating product recalls ((Milgrom and Roberts, 1986; Moorthy and Zhao, 2000), Thirumalai and Sinha (2011), (Lu and Navas, 2021; Mukherjee and Chauhan, 2021; Rubel, 2018; Rubel et al., 2011)). The case where the recall is not anticipated by a hazard-myopic manufacturer is embedded in our model and it will also be analyzed.

Our work answers the following research questions:

RQ1: What are the optimal quality, pricing and advertising policies in the pre-crisis, crisis and post-crisis period?

RQ2: How do stochastic parameters (crisis likelihood, recovery likelihood, crisis impact and recovery intensity) affect the firm's decisions and profit?

RQ3: Are hazard-myopic firms always worse off than the farsighted firms who anticipate a recall and a recovery?

RQ4: Does the time of recall play any role in determining the firm profitability?

Our findings underscore the importance of the stochastic model parameters, crisis likelihood (hazard rate of the process of occurrence of recall) and recovery likelihood (hazard rate of the process of occurrence of recovery) as well as the crisis impact (measured by the percentage drop of perceived quality) and recovery intensity (measured by the percentage growth of perceived quality). We found that a firm should strategically adjust pricing, advertising and design quality efforts depending on the above parameters and the decision regime. We also show that hazard myopia badly hurts a firm's profit in case of high-impact recalls. In fact, farsightedness can help a firm escape bankruptcy in case of high-impact product harm crisis. Moreover, recall time can significantly impact the profitability of myopic or farsighted firms.

While most of the modelling literature focus on either advertising, pricing or quality decisions, and the firm reactions in pre-crisis and post-crisis regimes, we contribute to the extant literature by articulating the interactive effect of several decisions and adding a third regime. The analysis, though complex, helps in drawing some useful insights and reconciling some conflicting findings in the literature. To capture the dynamics of decisions and the model parameters, we use an optimal control model. The investigation of the interaction of the above decisions is of utmost importance because when a product harm crisis recall happens, a firm usually makes multiple decisions, which are not static but depend on the information available to the firm at the moment. Therefore, such decisions will change over time with available information on, e.g., perceived quality of the brand/firm, effectiveness of advertising, consumer loyalty, recall costs, available budget and so on. In our study, an example of such information is advertising effectiveness on which the firm's optimal advertising will depend. Similarly, pricing and quality decisions will depend on several vital parameters and on the dynamic state - perceived quality. To the best of our knowledge, pricing, quality and advertising decisions as well as recall and recovery have not been examined simultaneously. Here lies our main contribution.

The rest of the paper is organized as follows. In Section 2, we discuss the literature and position our paper. We elaborate the model in Section 3, and discuss the analytically derived optimal policies in Section 4. Section 5 complements our analytical findings with numerical simulations to draw insightful results. In Section 6, we analyze the impact of the model parameters on firm profit. Finally, we conclude in Section 7 by summarizing our contribution and giving directions for future research.

2 Literature review

The product recall literature spans across marketing, supply chain/operations management and finance areas. A comprehensive literature review about the marketing tools and product harm crisis can be found in Cleeren et al. (2017). Our article belongs to the intersection of operations management (represented by product quality) and marketing (pricing and advertising decisions). Therefore, the following brief literature review is articulated around these three instruments.

Product recall and advertising. A vast literature in marketing and supply chain management investigates firm's strategies during a product recall. We review the closest contributions to ours. According to Dawar and Pillutla (2000), brand equity and credibility can be severely affected by a product recall, and advertising is an important toll to regain brand equity. Some studies conclude that advertising should be boosted in the post-crisis regime to mitigate the damage caused due to lack of consumer's trust or reliability of the brand (Borah and Tellis, 2016; Cleeren et al., 2008, 2013; Eilert, 2013; Rubel et al., 2011; Van Heerde et al., 2007). Gao (2015) and Mukherjee and Chauhan (2021) found that advertising level should depend on the impact of the crisis. In addition, the effectiveness of advertisement is brand-dependent. Indeed, Cleeren et al. (2008) concludes that pre-crisis brand advertising can act as a buffer to minimize the loss due to product recalls and this buffering effect is stronger for dominant brands. During product recall, similar result is showed by Van Heerde et al. (2007), concretely, advertisement by a dominant brand can be more effective than by a smaller brand. Zhao et al. (2011) found that advertising effectiveness, generally, substantially decreases by 43% during a recall comparatively to pre-crisis.

Product recall and pricing. Like advertising, pricing is part of the arsenal used to attenuate the effects of product recall. The question is essentially whether to change the price after a recall, either increasing or decreasing it, or keeping it at its pre-crisis level. Decreasing the price can incentivize price-sensitive consumers to purchase products of the same recalled brand. However, it may also be perceived as a signal that the firm is less self confident in its products. Marn et al. (2003) states that some managers decide to raise the price to increase revenue when demand plummets. Studying 60 product harm crisis incidents, Cleeren et al. (2013) obtain that both strategies have been adopted by firms during product recalls. Considering price competition, Rubel (2018) shows that price of high-risk recall products should be higher than risk-free products, with the price level depending on whether the firm is a market leader. However, under an impending recall, a market leader firm may not be more profitable.

The pricing decisions may depend on the price sensitivity of the product. While Van Heerde et al. (2007) finds that price sensitivity will depend on the brand, Zhao et al. (2011) finds that consumers become less sensitive during a product harm crisis and focus more on perceived quality, which is the next item.

Product recalls and quality. As one could expect, a recall affects negatively consumer's perception of product quality (Crafton et al., 1981; Reilly and Hoffer, 1983), and has considerable financial implications. Indeed, the firm incurs a series of costs, e.g., litigation, handling returns, disposal, fixing defective items, lost sales revenue and so on.[1] Also, quality failure may induce some long-term costs due to the diminishing reputation of the firm and consumer loyalty (Chao et al., 2009; Gao, 2015; Hendricks and Singhal, 2001; Thirumalai and Sinha, 2011).

Product quality is related to the two other instruments mentioned above. In a study, Kirmani and Rao (2000) show that price can be a signal of quality, which attracts quality-sensitive consumers. Wolinsky (1983) concludes that product prices help in differentiating product quality levels. Moorthy and Zhao (2000) investigated 162 brands and found that advertising expenditure positively affects perceived quality while considering the simultaneous effects of objective quality, price, and market share. However, the effectiveness of price and advertising on perceived quality will be moderated by consumer's previous experience. Looking at the joint role of advertising and quality, Lu and Navas (2021) finds that the pre-crisis quality improvement not only results in high pre-crisis goodwill build-up, but also helps the recovery in post-crisis regime.

Whatever the decisions retained in the different studies, they must be adapted over time. The literature either focused on the pre-crisis and post-crisis periods (Cleeren et al., 2013; Mukherjee and Chauhan, 2021; Rubel, 2018; Rubel et al., 2011), or retained three periods, namely, pre-crisis, crisis and post-crisis (Van Heerde et al., 2007; Zhao et al., 2011).

Our model is crafted from the above findings and assumes (i) that perceived quality is affected by advertising, price and objective (design) quality, and (ii) that a right management of advertising, price and perceived quality can help alleviating the harmful effects of a product recall. The decisions need to be adapted to the period in which the firm finds itself. During the pre-crisis period, the firm makes its decisions taking into account that a recall may happen. During the crisis period, some changes are required to internalize the loss of reputation, perceived quality and advertising effectiveness. Advertising, pricing and investment in quality have again to be adapted during the recovery period (post-crisis period), which can be full or partial. We extend the literature by building an optimal control model and characterizing the optimal advertising, pricing and investment in quality decisions during the three time windows. In particular, we show how the firms should react optimally when the product recall and the recovery happen at random times.

[1] A comprehensive list of recall costs can be found here: https://cdn2.hubspot.net/hub/288450/file-606071909-pdf/Capturing_recall_costs.pdf

3 The model

Consider a manufacturer selling a single product over time $t \in [0, +\infty)$. Denote by $i \in \{1, 2, 3\}$ the regime under which the firm operates, with $1, 2$ and 3 corresponding to pre-crisis, crisis and post-crisis regime, respectively. The demand depends negatively on the price $p_i(t)$ and positively on the product quality $Q(t)$, which is a state variable. The demand is given by the following linear function:

$$D_i(t) = \alpha - \beta p_i(t) + \gamma Q(t), \tag{1}$$

where α, β and γ are positive parameter. In this formulation, the market potential is given by α plus the variable term $\gamma Q(t)$, where γ measures the marginal impact of quality on demand. The parameter β is the price sensitivity. Our demand functional form is common in the economics, marketing and operations management literature and is micro-founded, that is, a linear demand can be derived from consumer maximization of a quadratic utility function.

The product quality is a construct that encompasses perceived quality, whose drivers are price and advertising, and objective quality, which refers to tangible features of the product. Denote by $a_i(t)$ the advertising effort at time t, and let the advertising cost be given by the convex increasing function $C_a(a_i(t)) = \frac{\mu_a}{2} a_i^2(t)$. This cost function is common in the dynamic advertising literature; see, e.g., the surveys in Huang et al. (2012), Zhang et al. (2013) and Jørgensen and Zaccour (2014). Let $q_i(t)$ be the investment made in quality at time t. The evolution of the total product quality is governed by the following linear-differential equation:

$$\dot{Q}(t) = k_i a_i(t) + l_i p_i(t) + m_i q_i(t) - \epsilon_i Q(t), \text{ and } Q(0) = Q_0, \tag{2}$$

where k_i, l_i and m_i are positive corresponding to the marginal impact of advertising, price and quality investment, respectively; $\epsilon_i > 0$ is the decay parameter and Q_0 the initial value of the product quality. In the above dynamics, the term $k_i a_i(t) + l_i p_i(t)$ measures the positive contribution of price and advertising to the evolution of the perceived quality, while $m_i q_i(t)$ accounts for the impact of design quality. We note that a series of papers also modelled quality as a state variable in either a firm optimization setting (see, e.g., Chand et al. (1996); De Giovanni (2020); Liu et al. (2015), or in a dynamic game (see, e.g., El Ouardighi and Kim (2010); El Ouardighi and Kogan (2013); Martín-Herrán et al. (2012) with the exception of the latter, where the state variable is perceived quality, the other contributions retained conformance and/or design quality.

The occurrences of the product recall and its subsequent recovery are captured by a stochastic process $R(t)$, where $R(t) = 1$ refers to non-crisis periods, i.e., before the recall and after recovery, and $R(t) = 2$ to the crisis period. The process of recall occurrence is given by:

$$\begin{aligned} \lim_{dt \to 0} \frac{P[R(t+dt) = 2 | R(t) = 1]}{dt} &= \lambda, \\ \lim_{dt \to 0} \frac{P[R(t+dt) = 1 | R(t) = 2]}{dt} &= \chi. \end{aligned} \tag{3}$$

The recall and recovery times are t_1 and t_2, respectively, and are exponentially distributed. The hazard rates λ and χ are constants. The exponential hazard rate assumption follows Rubel et al. (2011) and Rubel (2018). The hazard rate is not controlled or managed by the firms during the lifetime of the product and hence may remain constant. Empirical research shows that such rate is determined by causes which are not observable before the realization of the defect (Thirumalai and Sinha, 2011). The three regimes are characterized as follows:

Pre-crisis regime: The recall has not yet happened, but the manufacturer takes into account that it may occur in the future.

Crisis regime: The manufacturer has issued a recall at the random time t_1 and is incurring recall costs. The firm is also suffering from loss of quality, mainly due to change in consumer's perception of quality (Reilly and Hoffer, 1983; Thirumalai and Sinha, 2011), loss in advertising effectiveness ($k_2 < k_1$) (Liu and Shankar, 2015; Van Heerde et al., 2007), lower impact of price on quality ($l_2 < l_1$) and a possible increase in the decay rate ($\epsilon_2 > \epsilon_1$) (Mukherjee and Chauhan, 2021). The loss in quality level is given by a jump in the state value, i.e., $Q(t_1^+) = (1-\omega)Q(t_1^-)$, with $\omega \in [0,1]$. The cost of recall is given by

$$\mathrm{CR} = \rho c_r \int_0^{t_1} D(t)dt,$$

where ρ is the proportion of defective items (or fraction of recalled products) sold during the pre-crisis period, and c_r is the unit recall cost (Chao et al., 2009). Although t_1 is random at the beginning of the planning horizon, it is deterministic when the recall costs are accounted for in the second regime.

Post-crisis regime: The post-crisis regime begins at a random time t_2. The firm has recovered from crisis and may observe a positive jump in perceived quality given by $Q(t_2^+) = (1+\psi)Q(t_2^-)$, where $0 \leq \psi \leq \omega < 1$. Also, the manufacturer enjoys an increase in advertising effectiveness ($k_3 = k_1 > k_2$) and in price impact on perceived quality ($l_3 = l_1 > l_2$).

Summarizing the above description, (2) can be rewritten in detailed form as follows:

$$\dot{Q}(t) = \begin{cases} k_1 a_1(t) + l_1 p_1(t) + m_1 q_1(t) - \epsilon_1 Q(t), & t \in [0, t_1], \quad Q(0) = Q_0, \\ k_2 a_2(t) + l_2 p_2(t) + m_2 q_2(t) - \epsilon_2 Q(t), & t \in (t_1, t_2], \quad Q(t_1^+) = (1-\omega)Q(t_1^-), \\ k_3 a_3(t) + l_3 p_3(t) + m_3 q_3(t) - \epsilon_3 Q(t), & t \in (t_2, \infty), \quad Q(t_2^+) = (1+\psi)Q(t_2^-). \end{cases}$$

We assume that the quality cost is the sum of quality conformance cost and the quality improvement cost. Denote by ϕ the maximum hazard rate beyond which firm stops production and let θ be a positive scaling parameter. The quality cost is specified as follows

$$C_q(q_i(t)) = \begin{cases} \frac{\mu_q + \theta(\phi - \lambda)}{2} q_i^2(t), & \text{for } i = 1, \\ \frac{\mu_q + \theta\phi}{2} q_i^2(t), & \text{for } i = 2, 3, \end{cases}$$

where $\frac{\mu_q}{2} q_i^2(t)$ is the quality improvement cost, which is the same in all periods. The term $\frac{\mu_q + \theta(\phi - \lambda)}{2} q_1^2(t)$ reflects the idea that the conformance quality cost increases with the lower value of crisis likelihood λ. The maximum quality cost $\frac{\mu_q + \phi}{2} q_1^2(t)$ materializes when $\lambda = 0$, that is, there is no recall and the firm fully conforms to product quality. While the assumption that quality impacts hazard rate will take us into technical complexities, this approach can still help in capturing the effect of quality on hazard. The costs structure highlights that the firm knows that if it invests more, then the likelihood of a recall will be lower. Nevertheless, this approach does not bring in additional technical difficulties. Notably, in the second and third periods there is full conformance of quality as only one recall occurs in the planning horizon. Therefore, the conformance cost in the second and third regimes are maximal.

All the parameters and their ranges are recalled in Table 1.

3.1 The manufacturer's optimization problem

Denote by $\pi_i(t)$ the instantaneous profit at time t in regime i, that is,

$$\pi_i(t) = p_i(t) D_i(t) - C_a(a_i(t)) - C_q(q_i(t)).$$

To define the manufacturer's overall optimization problem, we proceed backward.

Post-crisis regime's problem

Given that the recovery period starts at t_2, and assuming that no further recall can occur (Rubel et al., 2011), the total discounted profit on $[t_2, \infty)$ is given by $J_3 = \displaystyle\int_{t_2}^{\infty} e^{-rt} \pi_3(t)\, dt$. The third regime's

Table 1: Parameter definitions

Parameters	Description	Variation with regime
$\alpha \in [0, \infty)$	Market potential	No
$\beta \in [0, \infty)$	Price sensitivity of demand	No
$\gamma \in [0, \infty)$	Quality level sensitivity of demand	No
$\mu_a \in [0, +\infty)$	Proportionality constant for advertising cost	No
$\mu_q \in [0, +\infty)$	Proportionality constant for quality cost	No
$\theta \in [0, +\infty)$	Proportionality constant for maximum hazard rate	No
$\phi \in [0, 2]$	Maximum hazard rate beyond which firm stops production	No
$\rho \in [0, 1]$	Proportion of defective items	No
$\lambda \in [0, +\infty)$	Crisis likelihood	NA
$\omega \in [0, 1]$	Impact of crisis on quality level	NA
$\chi \in [0, +\infty)$	Recovery likelihood	NA
$\psi \in [0, 1]$	Impact of recovery on quality level	NA
$k_i \in [0, +\infty)$	Advertising effectiveness	Yes
$l_i \in [0, +\infty)$	Price signal of quality	Yes
$m_i \in [0, +\infty)$	Sensitivity of design quality on quality level	Yes
$\epsilon_i \in [0, 1)$	Decay of quality level	Yes

profit-maximization problem of the manufacturer is as follows:

$$V_3 = \max_{a_3, p_3, q_3} \int_{t_2}^{\infty} e^{-rt} \pi_3(t)\, dt, \tag{4}$$

$$\text{Subject to}$$

$$\dot{Q}(t) = k_3 a_3(t) + l_3 p_3(t) + m_3 q_3(t) - \epsilon_3 Q(t), \text{ and } Q(t_2^+) = (1+\psi) Q(t_2^-).$$

Denote by $V_3((1+\psi)Q)$ the solution to the above optimization problem, which depends on the value on quality at initial time t_2^+. The term $V_3((1+\psi)Q)$ plays the role of a salvage value in period 2's optimization problem.

Crisis regime's problem

In the crisis period the manufacturer knows that the recall cost incurred in the second period is $\int_0^{t_1} \rho c_r D_1(t) dt$. Therefore, the crisis regime's instantaneous profit π_2 is given by:

$$\pi_2 = D_2(t) p_2(t) - \frac{\mu_a}{2} a_2^2(t) - \frac{\mu_q + \theta\phi}{2} q_2^2(t) - \int_0^{t_1} \rho c_r D_1(t) dt.$$

In the above equation, the integrand $\int_0^{t_1} \rho c_r D_1(t) dt$ is a constant because the demand D_1 and the proportion of defective items ρ have realized and are known. (c_r is the constant average unit recall cost incurred in the second regime.) In the crisis regime, the manufacturer does not know the random time $t = t_2$ at which recovery will occur but the manufacturer anticipates the recovery with intensity ψ and hazard rate χ (which follows an exponential distribution). Therefore, the expected profit at the beginning of the crisis regime can be defined as follows:

$$\begin{aligned} J_2 &= \mathbb{E}_t \left[\int_{t_1}^{t} e^{-rs} \pi_2(s) ds + e^{-rt} J_3 \right], \\ &= \int_0^{\infty} \left(\int_{t_1}^{t} e^{-rs} \pi_2(s) ds + e^{-rt} J_3 \right) \chi e^{-\chi t} dt, \\ &= \int_{t1}^{\infty} \left(\chi e^{-\chi} \int_{t_1}^{t} e^{-rs} \pi_1(s) ds \right) dt + \chi \int_{t1}^{\infty} e^{-(r+\chi)t} J_3 dt. \end{aligned} \tag{5}$$

To evaluate the first right-hand side term, we integrate by parts. Let $U = \int_{t1}^{t} e^{-rs}\pi_2(s)ds$ and $V = -e^{-\chi t}dt$. Therefore, we have $dU = e^{-rt}\pi_2(t)dt, dV = \chi e^{-\chi t}$ and

$$\int_{t1}^{\infty}\left(\int_{t1}^{t} e^{-rt}\pi_2(s)ds\right)\chi e^{-\chi t}dt = (-Ue^{-\chi t})|_{t_1}^{\infty} - \int_{t_1}^{\infty}(e^{-\chi t})(e^{-rt}\pi_2(t))dt,$$
$$= 0 + \int_{t1}^{\infty} e^{-(r+\chi)t}\pi_2(t)dt. \quad (6)$$

Therefore, from (5) and (6) the second regime's profit is

$$J_2 = \int_{t1}^{\infty} e^{-(r+\chi)t}\pi_2(t)dt + \chi\int_{t1}^{\infty} e^{-(r+\chi)t}J_3dt,$$
$$= \int_{t1}^{\infty} e^{-(r+\chi)t}\left(\pi_2(t)dt + \chi J_3\right)dt. \quad (7)$$

Consequently, the manufacturer's profit maximizing problem in the crisis regime is given by

$$V_2 = \max_{a_2,p_2,q_2}\int_{t_1}^{\infty} e^{-(r+\chi)t}\Big(D_2(t)p_2(t) - \int_0^{t_1}\rho c_r D(s)ds - \frac{\mu_a}{2}a_2^2(t) - \frac{\mu_q+\theta\phi}{2}q_2^2(t)$$
$$+ \chi V_3((1+\psi)Q)\Big)dt \quad (8)$$

Subject to

$$\dot{Q}(t) = k_2a_2(t) + l_2p_2(t) + m_2q_2(t) - \epsilon_2Q(t), \text{ and } Q(t_1^+) = (1-\omega)Q(t_1^-).$$

Note that in (8), J_3 has been replaced by V_3 which is the maximized third regime's profit.

Pre-crisis regime's problem

In the pre-crisis regime the manufacturer anticipates the recall at the random time $t = t_1$, which follows an exponential distribution with parameter λ. In this period, the manufacturer's instantaneous profit is given by:

$$\pi_1(t) = D_1(t)p_1(t) - \frac{\mu_a}{2}a_1^2(t) - \frac{\mu_q+\theta(\phi-\lambda)}{2}q_1^2(t).$$

Proceeding in the same manner as for the crisis regime's problem, the manufacturer's long-term expected profit at the beginning of the planning horizon is

$$J_1 = \mathbb{E}_t\left(\int_0^t e^{-rs}\pi_1(s)ds + e^{-rt}\left(J_2 - \rho c_r D_1(t)\right)\right)$$
$$= \int_0^{\infty}\left(\int_0^t e^{-rs}\pi_1(s)ds + e^{-rt}\left(J_2 - \rho c_r D_1(t)\right)\right)\lambda e^{-\lambda t}dt$$
$$= \int_0^{\infty}\left(\lambda e^{-\lambda}\int_0^t e^{-rs}\pi_1(s)ds\right)dt + \lambda\int_0^{\infty} e^{-(r+\lambda)t}J_2dt - \rho c_r\int_0^{\infty}\lambda e^{-(\lambda+r)t}D_1(t)dt$$
$$\triangleq I_1 + \lambda I_2 - \rho c_r I_3. \quad (9)$$

To evaluate the three integrals, we integrate by parts. Starting by I_1, letting $U = \int_0^t e^{-rs}\pi_1(s)ds$ and $V = -e^{-\lambda t}$, we obtain $dU = e^{-rt}\pi_1(t)dt$, $dV = \lambda e^{-\lambda t}$ and

$$I_1 = \int_0^{\infty}\left(\int_0^t e^{-rt}\pi_1(s)ds\right)\lambda e^{-\lambda t}dt,$$
$$= (-Ue^{-\lambda t})|_0^{\infty} - \int_0^{\infty}(e^{-\lambda t})(e^{-rt}\pi_1(t))dt,$$
$$= 0 + \int_0^{\infty} e^{-(r+\lambda)t}\pi_1(t)dt.$$

The above results follow from $(-Ue^{-\lambda t})|_0^\infty = 0$ since $\lim_{t\to\infty} e^{-\lambda t} = 0$ and $U(0) = \int_0^0 e^{-rs}\pi_1(s)ds = 0$. Similarly, we have

$$I_3 = \int_0^\infty e^{-(r+\lambda)t} D_1(t)dt.$$

Therefore, (9) becomes

$$\begin{aligned} J_1 =& I_1 + \lambda I_2 - \rho c_r I_3 \\ =& \int_0^\infty e^{-(r+\lambda)t}\pi_1(t)dt + \lambda \int_0^\infty e^{-(r+\lambda)t} J_2 dt - \lambda\rho c_r \int_0^\infty e^{-(r+\lambda)t} D_1(t)dt \\ =& \int_0^\infty e^{-(r+\lambda)t}\left(\pi_1(t) + \lambda(J_2 - \rho c_r D_1(t))\right) dt. \end{aligned}$$

Consequently, the overall optimization problem is as follows:

$$\begin{aligned} V_1 =& \max_{a_1,p_1,q_1} \int_0^\infty e^{-(r+\lambda)t}\Big(D_1(t)p_1(t) - \frac{\mu_a}{2}a_1^2(t) - \frac{\mu_q + \theta(\phi-\lambda)}{2}q_1^2(t) \\ & + \lambda V_2((1-\omega)Q - \rho c_r D_1(t))\Big)dt \qquad (10) \\ & \text{Subject to} \\ \dot{Q}(t) =& k_1 a_1(t) + l_1 p_1(t) + m_1 q_1(t) - \epsilon_1 Q(t), \text{ and } Q(0) = Q_0. \end{aligned}$$

Given the linear-quadratic structure of the dynamic optimization problem, we make the informed guess that the value function is quadratic in $Q(t)$. More specifically, the value functions in the three regimes are written as follows:

$$\begin{aligned} V_1(Q) =& X_1 Q^2 + X_2 Q + X_3, \\ V_2(Q) =& Y_1 Q^2 + Y_2 Q + Y_3, \\ V_3(Q) =& Z_1 Q^2 + Z_2 Q + Z_3, \end{aligned}$$

where the coefficients X_j, Y_j, and $Z_j, j = 1, \ldots, 3$ are to be determined.

4 Optimal policies

Following (Mukherjee and Chauhan, 2021), we classify the firms into two categories - firms that envision the recall ex-ante, which are referred to as *farsighted firms*, and those that are "surprised" by the recall, that is, *hazard myopic firms*. Next, we provide the optimal policies for each category. The proofs for the propositions presented in this section can be found in Appendix.

4.1 Optimal policies for the farsighted firm

In this section, we present the optimal advertising, pricing and quality efforts of the firm. As a standard approach, we solve the Hamilton-Jacobi-Bellman (HJB) equations to obtain the solution of the decision problems of the manufacturer in the three regimes, starting from the last one. To simplify the presentation, we shall drop from now on the time argument, unless an ambiguity may arise.

The HJB equation of the third period is:

$$rV_3(Q) = \max_{a_3,p_3,q_3}\Big[p_3(\alpha - \beta p_3 + \gamma Q) - \frac{\mu_a}{2}a_3^2 - \frac{\mu_q + \theta\phi}{2}q_3^2 + \frac{\partial V_3}{\partial Q}(k_3 a_3 + l_3 p_3 + m_3 q_3 - \epsilon_3 Q)\Big]. \quad (11)$$

The HJB equation of the second period is given by:

$$(r+\chi)V_2(Q) = \max_{a_2,p_2,q_2}\Big[p_2(\alpha - \beta p_2 + \gamma Q) - \frac{\mu_a}{2}a_2^2 - \frac{\mu_q + \theta\phi}{2}q_2^2 - \rho c_r \int_0^{t_1} D_1(s)ds +$$

$$\frac{\partial V_2}{\partial Q}(k_2a_2 + l_2p_2 + m_2q_2 - \epsilon_2 Q) + \chi V_3((1+\psi)Q)\Big]. \tag{12}$$

Although the product recall occurs at a random time t_1, this date and $D_1(t)$ are known in the second regime. Consequently, the recall cost $\rho c_r \int_0^{t_1} D_1(s)ds$ is also a constant in this regime.

In the first regime, the HJB equation is given by:

$$(r+\lambda)V_1(Q) = \max_{a_1,p_1,q_1}\Big[p_1(\alpha - \beta p_1 + \gamma Q) - \frac{\mu_a}{2}a_1^2 - \frac{\mu_q + \theta(\phi-\lambda)}{2}q_1^2 + \frac{\partial V_1}{\partial Q}(k_1a_1 + l_1p_1 + m_1q_1 - \epsilon_2 Q) + \lambda(V_2((1-\omega)Q) - c_r\rho D_1)\Big]. \tag{13}$$

The following proposition characterizes the optimal policy in the three regimes.

Proposition 1. *Assuming an interior solution, the optimal advertising, pricing and quality policies are as follows:*

- *Pre-crisis regime:*

$$a_1 = \frac{k_1(2QX_1 + X_2)}{\mu_a}, \tag{14a}$$

$$p_1 = \frac{\alpha + \beta\lambda\rho c_r + l_1X_2 + (2l_1X_1 + \gamma)Q}{2\beta}, \tag{14b}$$

$$q_1 = \frac{m_1(2QX_1 + X_2)}{\theta(\phi-\lambda) + \mu_q}, \tag{14c}$$

  where $X_j, j = 1, \ldots, 3$ are the coefficients of the value function $V_1(Q) = X_1Q^2 + X_2Q + X_3$ and are defined in the Appendix.

- *Crisis regime:*

$$a_2 = \frac{k_2(2QY_1 + Y_2)}{\mu_a}, \tag{14d}$$

$$p_2 = \frac{\alpha + l_2Y_2 + (2l_2Y_1 + \gamma)Q}{2\beta}, \tag{14e}$$

$$q_2 = \frac{m_2(2QY_1 + Y_2)}{\theta\phi + \mu_q}, \tag{14f}$$

  where $Y_j, j = 1, \ldots, 3$ are the coefficients of the value function $V_2(Q) = Y_1Q^2 + Y_2Q + Y_3$ and are defined in the Appendix.

- *Post-crisis regime:*

$$a_3 = \frac{k_3(2QZ_1 + Z_2)}{\mu_a}, \tag{15a}$$

$$p_3 = \frac{\alpha + l_3Z_2 + (2l_3Z_1 + \gamma)Q}{2\beta}, \tag{15b}$$

$$q_3 = \frac{m_3(2QZ_1 + Z_2)}{\theta\phi + \mu_q}, \tag{15c}$$

  where $Z_j, j = 1, \ldots, 3$ are the coefficients of the value function $V_3(Q) = Z_1Q^2 + Z_2Q + Z_3$.

We make the following comments:

1. The expressions of X_j, Y_j and Z_j are very long and do not provide any direct qualitative insight. Therefore, they are not reported. However, we highlight that Z_1, Z_2 and Z_3 are independent of all the stochastic parameters because in the post-crisis period the recall and recovery have already been realized.

2. The policies share some features. First, they are linear in the state variable, a result that is reminiscent to the linear-quadratic structure of the dynamic optimization problem. Second, although the expressions of the optimal policies during the crisis and post-crisis periods look similar, we note that the Y_js coefficients depend on the stochastic recovery rate χ and on the recovery intensity ψ (see Appendix). This is because the firm anticipates during the crisis period that the the recovery occurs in the post-crisis period. Moreover, since the firm foresees the recovery, χ and ψ also have indirect influence on the X_js. In fact, the differences in the policies during the crisis and the post-crisis periods are due to (i) the influence of the parameters ξ and χ on Y_j, and (ii) the jump $Q(t_2^+) = (1+\psi)Q(t_2^-)$ that occurs at t_2, which is the random time of recovery. Finally, the stochastic parameter λ and the recall impact ω affect the coefficients X_js directly.
3. In each regime, the advertising and quality policies are determined by the familiar rule of marginal cost equals marginal revenue. Indeed, the advertising policy can be rewritten as

$$\begin{aligned}\mu_a a_i &= k_i\left(2Q\Gamma_1+\Gamma_2\right) \Leftrightarrow C_a'\left(a_i\right)=\frac{\partial \dot{Q}}{\partial a_i}\cdot V_i'\left(Q\right)\\ &\Leftrightarrow \text{ marginal cost} = \text{ (marginal impact of } a_i \text{ on } \dot{Q}) \times \text{(marginal payoff-to-go)},\end{aligned}$$

where $(\Gamma_1,\Gamma_2)\in\{(X_1,X_2),(Y_1,Y_2),(Z_1,Z_2)\}$ and for $i=1,2,3$. Similarly, for quality we have

$$\begin{aligned}\left(\theta(\phi-\lambda)+\mu_q\right)q_1 =& m_1(2Q\Gamma_1+\Gamma_2)\Leftrightarrow C_q'\left(q_1\right)=\frac{\partial \dot{Q}}{\partial q_1}\cdot V_1'\left(Q\right)\\ \left(\theta\phi+\mu_q\right)q_i =& m_i(2Q\Gamma_1+\Gamma_2)\Leftrightarrow C_q'\left(q_i\right)=\frac{\partial \dot{Q}}{\partial q_i}\cdot V_i'\left(Q\right) \text{ for } i=2,3.\end{aligned}$$

4. Comparatively to the pricing policy in the crisis and post-crisis regimes, the pricing policy in the pre-crisis regime has the additional term $\beta\lambda\rho c_r$, which is defined as follows:

$$\beta\lambda\rho c_r = \frac{\partial D}{\partial p}\cdot \text{crisis likelihood} \cdot \text{proportion of defective items} \cdot \text{unit recall cost}.$$

Put differently, this term accounts for the probable marginal future cost of recall to be incurred in the crisis period. Moreover, the pre-crisis quality effort contains the term $\theta(\phi-\lambda)+\mu_2$ in the denominator which is different from the expressions of quality efforts in the two other regimes.

4.2 Optimal policies of a hazard-myopic firm

In the previous subsection, we assumed that the firm is farsighted. A hazard-myopic firm overlooks the possibility of a recall but realizes its effect immediately after it happens (Mukherjee and Chauhan, 2021). In our model, the myopic firm fails to anticipate the recall as well as the recovery. Therefore, the firm optimizes its payoff function assuming an infinite planning horizon, without any discontinuity in the state variable. Once the recall or the recovery occurs, the firm observes the jump in state variable and adjusts its decisions.

The HJB equations of a myopic firm in period $i\in\{1,2,3\}$ is given by

$$rV_i(Q)=\max_{a_i,p_i,q_i}\left(p_i\left(\alpha-\beta p_i+\gamma Q\right)-\frac{\mu_a}{2}a_i^2-\frac{\mu_q}{2}q_i^2+\frac{\partial V_3}{\partial Q}(k_i a_i+l_i p_i+m_i q_i-\epsilon_i Q)\right). \tag{16}$$

Proposition 2. *For a hazard-myopic firm the optimal advertising, price and quality policies in regime $i\in\{1,2,3\}$ are given by*

$$a_i = \frac{k_1(2Q\Delta_{i1}+\Delta_{i2})}{\mu_a}, \tag{17}$$

$$p_i = \frac{\alpha+(2l_1\Delta_{i1}+\gamma)Q}{2\beta}, \tag{18}$$

$$q_i = \frac{m_1(2Q\Delta_{i1} + \Delta_{i2})}{\mu_q}, \tag{19}$$

where Δ_{i1}, Δ_{i2} *and* Δ_{i3} *are the coefficients of the value function* $V_i(Q) = \Delta_{i1}Q^2 + \Delta_{i2}Q + \Delta_{i3}$.

As in the farsighted scenario, the policies are linear and the value functions quadratic in the state variable Q for the same reason. Further, the decisions of the firm have similar structures in all three regimes, with however different Δ_js because of the jumps in quality at the instants t_1 and t_2. Also, these strategies can be interpreted along the same line as before. One major difference with respect to the farsighted firm is that the myopic firm value functions' coefficients do not incorporate any stochastic parameters χ and λ nor the impact parameters ω and ψ.

5 Numerical results

Our model has 26 parameters that can be regrouped as follows:

$$\begin{aligned} \text{Demand parameters} &: \alpha, \beta, \gamma, \\ \text{Cost parameters} &: \mu_a, \mu_q, \\ \text{Quality dynamics parameters} &: k_i, l_i, m_i, \epsilon_i, i = 1, \ldots, 3; Q_0, \\ \text{Recall and recovery related parameters} &: \theta, \phi, \lambda, \psi, \rho, \chi, c_r, \\ \text{Discount rate} &: r. \end{aligned}$$

To keep manageable the number of figures, and taking into account our research focus, we shall vary the values of the most important parameters, namely, λ, ω, χ and ψ, while fixing the following ones:[2]

$$\alpha = 25; \quad \beta = 2; \quad \gamma = 1; \quad \phi = 2; \quad \mu_a = \mu_q = \theta = 100; \quad m_i = 0.5; \quad Q_0 = 1.$$

Further, the retained values for parameters k_i, l_i and ϵ_i,which are assumed to depend on the impact of the recall (ω) and the magnitude of the recovery (ψ), are provided in Table 1.

Table 2: Parameter values vs recall impact

Impact of recall (ω)	Value of k_i	Value of l_i	Value of ϵ_i
High impact; $\omega = .6$	$k_1 = k_3 = .05; k_2 = .025$	$l_1 = l_3 = .05; l_2 = .025$	$\epsilon_1 = \epsilon_3 = .03; \epsilon_2 = .05$
Benchmark impact; $\omega \leq .3$	$k_1 = k_3 = .05; k_2 = .04$	$l_1 = l_3 = .05; l_2 = .04$	$\epsilon_1 = \epsilon_3 = .03; \epsilon_2 = .05$

As a benchmark case, we let $\lambda = \omega = \chi = \psi = 0.3$, and assess the impact of varying them on the optimal firm decisions and performance.

5.1 Optimal manufacturer's decisions

In the following claims, we summarize our main findings on pricing, advertising and quality decisions when the firm is farsighted.

Claim 1. Within each regime, the price, advertising and design quality investments are increasing over time (Figures 1 to 6 and 9 to 15).

To interpret this result, recall that advertising investment and quality effort affect positively the state variable (total quality), which, in turn, has a positive impact on demand. Price is different because it plays a dual role, that is, it boosts total quality, and has a negative impact on demand. Therefore, any variation in the price must take into account this trade-off. However, as the impact on quality has a carry over effect and higher quality leads to a durable increase in the market potential,

[2]The result for any feasible constellation of parameter values can be provided by the authors upon request.

the instantaneous negative impact of an increase in price on demand is mitigated. In fact, the price ends up here having a positive effect on demand. This phenomenon where demand increases with price is referred to as the Veblen effect in the literature (Chenavaz and Eynan, 2021).

In a nutshell, the firm's strategies follow the total quality trajectory, which is increasing over time, within each regime, due to the positive effect of price, advertising and design quality efforts (see Figures 7, 8, 16 and 17).

Claim 2. The higher the recall impact ω, the lower the price, advertising and quality effort in the pre-crisis regime (Figures 1, 3 and 5).

The result regarding quality is counter intuitive at first glance. Indeed, one would expect a firm to invest more in design quality if a high impact recall is envisioned. However, as seen above, quality effort, price and advertising have a positive impact on both total quality and demand. Now, the larger the sales during the pre-crisis period are, the higher the number of defective items to recall during the crisis regime. Also, the recall costs increase with the recall impact ω because we have assumed that the fraction ρ of defective items increases with ω. Consequently, adopting the strategy described in the claim is a smart defensive move to limit the expected recall costs during the crisis regime.

Figures 1, 3 and 5 allow to make three additional comments: First, in the pre-crisis regime, the impact of ω on quality and advertising is more pronounced than on price, most likely because of the opposite effect of price on quality and demand. Second, the same ordering in the trajectories is observed in the two other regimes, after a jump down at t_1 and jump up at t_2, which are due to the quality shocks. Third, we note that for a low impact recall ($\omega = .05$), the manufacturer will ensure highest quality efforts in the pre-crisis period, i.e., $q_1 > q_2 > q_3$. However, as recall impact increases ($\omega = .60$), the quality effort in the crisis regime is the highest and $q_2 > q_3 > q_1$. One explanation is that the firm needs to catch up or to compensate for the low investments in the pre-crisis period to avoid large liabilities in the crisis period.

Claim 3. The impact of the crisis likelihood λ is as follows (Figures 2, 4 and 6):

1. Price goes down during the crisis regime with respect to the pre-crisis period, before continuing its pre-crisis trajectory during the recovery period.
2. Advertising shifts up during the crisis regime with respect to the pre-crisis period, and again during the recovery period.
3. The firm invests heavily in quality in the pre-crisis regime when λ is high, and decreases this investment in the two other regimes. For low and medium λ, its main investment is done during the crisis period.

The reduction in price during the crisis is to compensate for the loss in perceived quality. Indeed, a higher price would harm further the demand during this period. In the same way, boosting advertising helps in rebuilding the firm's total quality. The most interesting, and notable, result is the third one. A high crisis likelihood (Figure 6) results in highest pre-crisis quality efforts. This underscores the importance of the firm's farsightedness. At $\lambda = 1.5$, a recall is almost inevitable. Therefore, the manufacturer invests more in pre-crisis quality with the goal of increasing the pre-crisis perceived quality, and avoiding large liabilities during the crisis period. However, here the impact of recall is benchmark ($\omega = .3$). For a lower or higher value of ω the above finding may change.

Claim 4. The impacts of recovery intensity ψ and recovery likelihood χ are (qualitatively) the same on price, advertising and quality effort. These impacts are as follows (Figures 9 to 15):

1. The higher the recovery intensity and likelihood, the higher the price, advertising and quality effort in all regimes.
2. Price goes down during the crisis regime with respect to the pre-crisis period, and shifts up during the recovery period.
3. Advertising and quality are higher during the crisis regime than during the pre-crisis regime.

The results in this claim are intuitive; when the conditions are favorable, that is, the recovery intensity and recovery likelihood are high, then the firm charges a high price and invests more in advertising and quality to boost the long-term quality and, consequently, demand and profit. The rationale behind the shift down in price during the crisis is a before, that is, to avoid harming further demand. The opposite holds true for quality and advertising investments that are badly needed during the crisis period.

Claim 5. The parameters ω, λ, ψ or χ have no impact on the firm's policies during the recovery period (all figures, except the ones related to the quality level Q_i).

From these figures, we see some differences in price, advertising and quality when varying either ω, λ, ψ or χ. However, we must keep in mind the magnitude of this impact. Indeed, multiplying by 30 any of these parameters from 0.05 to 1.5, that is, an increase of $3,000\%$, leads to a variation in price, advertising or quality effort of less than 10%. Hence, the statement in the claim. This result is intuitive as the crisis is behind the firm and it knows (by assumption) that no other crisis can unfold.

For completeness, we show in Figures (7, 8, 16, 17) the evolution of the total quality over time (state variable) for different values of ω, λ, ψ or χ. The results are simply a compact presentation of what we obtained for the three policy instruments. To illustrate, quality level decreases with the recall impact ω in the three regimes, with the jump at $t_1$being higher as the recall impact increases. This is because all the decisions price, advertising and quality decrease with ω. We refrain from repeating the interpretations given before.

Before moving to profit considerations, let us wrap up our results so far: (i) The price goes down during the crisis period for the main important parameters related to product recall (ω, λ, ψ and χ), and so does the total quality. (ii) The investments in advertising and conformance quality go up during the crisis. (iii) Within each regime, the optimal policy is to practice price penetration, i.e., increase price over time, and to increase spendings on advertising and quality effort.

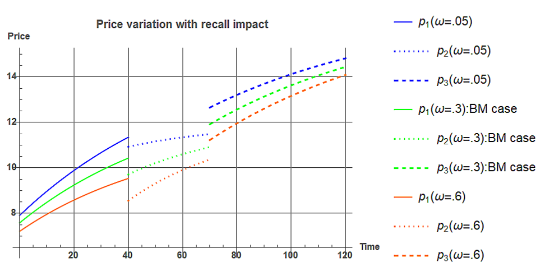

Figure 1: Price vs recall impact

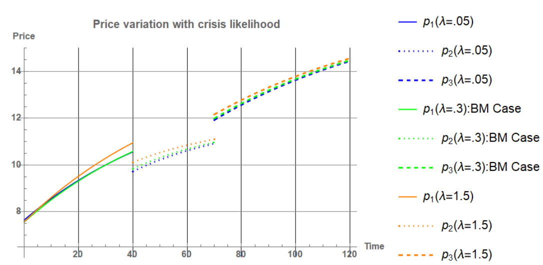

Figure 2: Price vs crisis likelihood

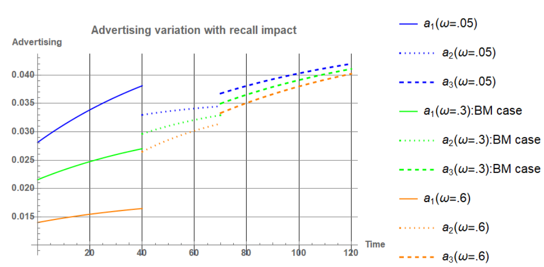

Figure 3: Advertising vs recall impact

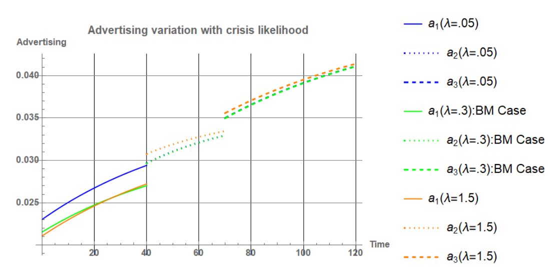

Figure 4: Advertising vs crisis likelihood

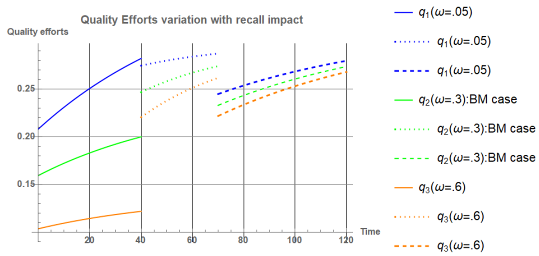

Figure 5: Quality efforts vs recall impact

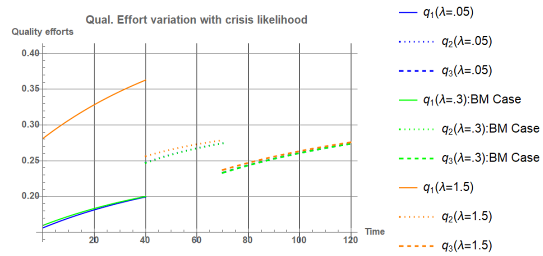

Figure 6: Quality efforts vs crisis likelihood

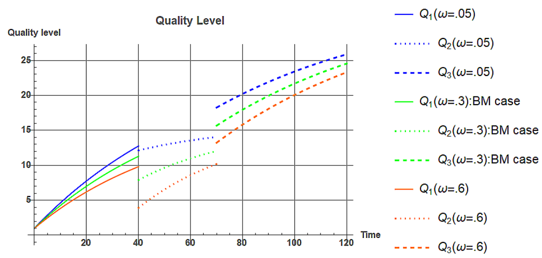

Figure 7: Quality level & recall impact

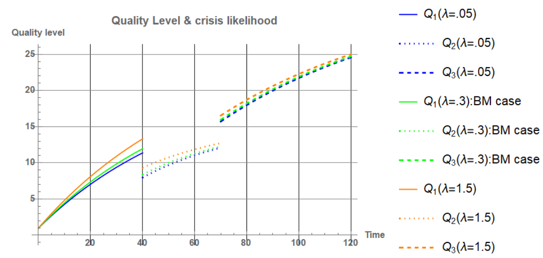

Figure 8: Quality level & crisis likelihood

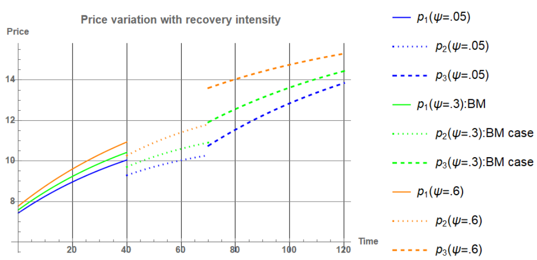

Figure 9: Price vs recovery intensity

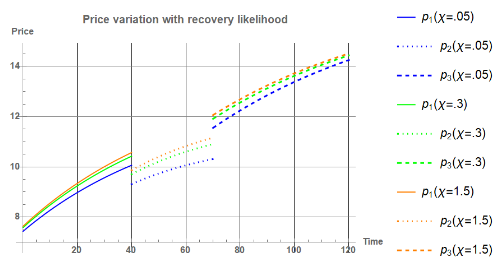

Figure 10: Price vs recovery likelihood

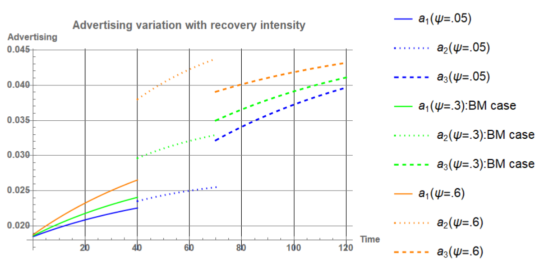

Figure 11: Advertising vs recovery intensity

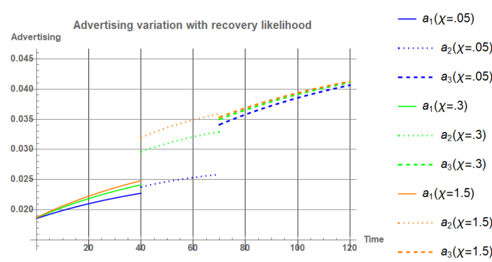

Figure 12: Advertising vs recovery likelihood

Figure 13: Advertising vs recovery intensity

5.2 Firm performance

The value functions give the maximized expected profit for each possible value of the state variable. We analyze how the value functions vary with our four focal parameters (ω, λ, ψ or χ) and next look at the interesting cases of bankruptcy and hazard myopia.

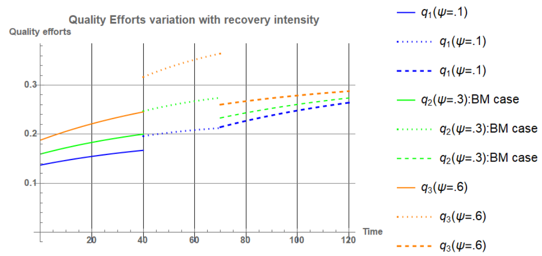

Figure 14: Quality efforts vs recovery intensity

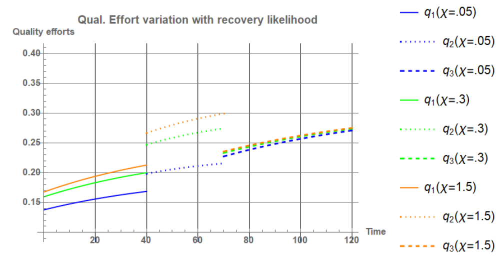

Figure 15: Quality efforts vs recovery likelihood

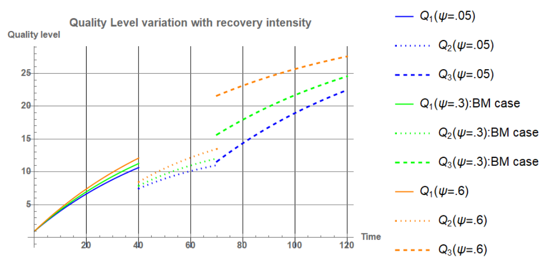

Figure 16: Quality level & recovery intensity

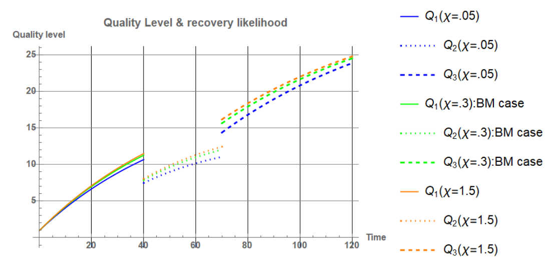

Figure 17: Quality level & recovery likelihood

The salient features of the value functions variations over time are:

- Without much surprise, we obtain that the optimal expected profit increases with the intensity of recovery and likelihood of recovery and decreases with the impact of the recall (see Figures 18 and 21).
- Surprisingly, the profit increases with the recall likelihood. Rubel (2018) obtained the same result in a duopoly competitive model, but the opposite for a monopoly. Though ours is a monopoly case, we still find that profit will increase with the crisis likelihood because of recovery. In presence of a substantial recovery, the firm uses higher levels of pricing, advertising and quality for higher likelihood of recall. Consequently, the profit soars. Things can be very different if the anticipated recovery is not promising. We will come back to this when discussing bankruptcy (see Figures 22 and 23).
- The profit sharply increases with the recovery likelihood in the approximate range initially. Thereafter, the increase in likelihood has little effect on the profit.

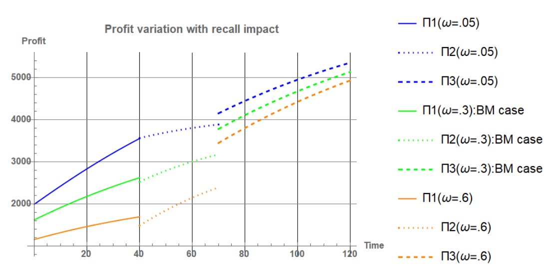

Figure 18: Value functions vs recall impact

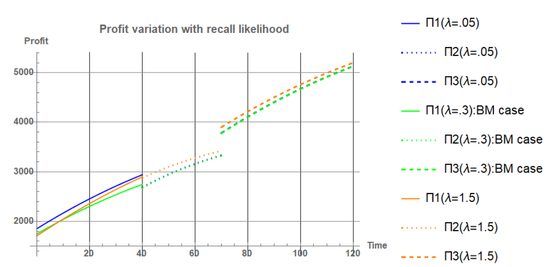

Figure 19: Value functions vs recall likelihood

The above results are obtained by varying one of the significant parameters (χ, ψ, ω and λ) at a time. Clearly, changing more than one parameter at a time may affect the outcomes. We exemplify two interesting cases: (i) when the firm might not manufacture the product at all while anticipating a recall; and (ii) when the firm becomes bankrupt after the recall.

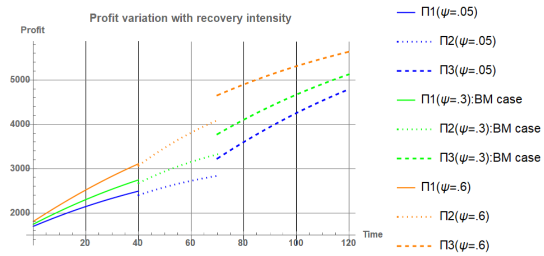

Figure 20: Value functions vs recovery intensity

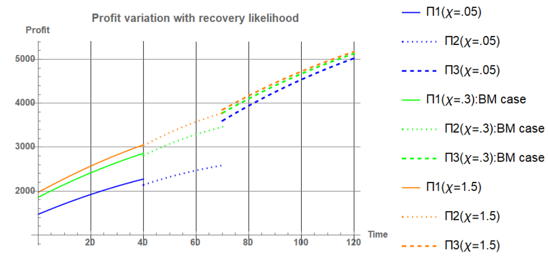

Figure 21: Value functions vs recovery likelihood

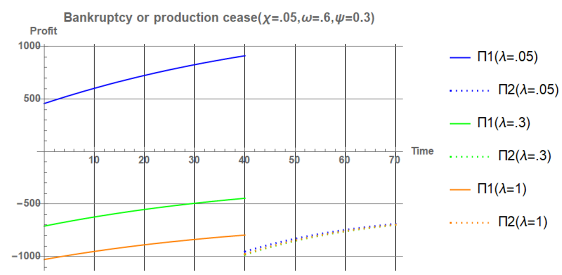

Figure 22: Bankruptcy

From the Figure 22, we note that in case of a very high impact recall ($\omega = .6$), when recall likelihood is low, the firm may still sell the product in the market but can go bankrupt as the second regime's value function becomes negative. As the likelihood of the recall increases, the pre-crisis profit becomes negative which shows that the firm might stop selling the product if it senses that the likelihood of a severe impact recall is high. This underscores the importance of anticipation of the recall. A myopic firm, on the other hand would have gone forward with the product sales and become bankrupt in the second regime.

As a high perceived quality may protect a firm from recall impact, can a high initial quality, seen as an insulation effect, protect a firm from bankruptcy? We capture this insulation effect by choosing an initial quality $Q_0 = 25$, as compared to $Q_0 = 1$ in the preceeding experiments. Figure 23 shows that in case of a high impact recall ($\omega = .6$ in this particular case), a firm may not be bankruptcy proof by the virtue of a high initial quality, assuming that recovery intensity and likelihood are low ($\chi = \psi = .05$). However, the threshold value of the impact beyond which the firm becomes bankrupt will increase when the initial perceived quality is higher. Grunwald and Hempelmann (2010) find that a highly reputed pre-crisis product quality may not be very effective in protecting the same in the crisis period. This is because the consumer's perception of the problem severity is not moderated by past quality. This supports our assumption that a high initial value of $Q(t_1^-)$ does not mean that the impact ω will assume a lower value.

Now, we compare the profit of a hazard myopic firm with that of a farsighted firm. In Figure 24, $M\Pi i$ and $FS\Pi i$ represent the profit of the myopic and farsighted firms in the i^{th} regime, respectively. In case of a very low impact recall, the myopic firm may be more profitable than the farsighted firm in the pre-crisis and post-crisis regime, but it is always worse off in the crisis regime. This finding is similar to the one in Mukherjee and Chauhan (2021) where the authors consider a competitive scenario. In the case of high impact recall, a myopic firm can become bankrupt after the recall, whereas the farsighted firm continues to make profit.

Finally, to look at how the profits vary with the time of the recall, we compare two cases of an early recall ($t_1 = 20$) and a late recall ($t_1 = 70$). The two main takeaways are as follows:

- For a myopic firm, an early recall is more beneficial than a late recall when the recall impact is low but a late recall is more profitable than an early recall if the recall impact is high (see Figures 26 and 27).
- On the other hand, a farsighted firm would prefer an early recall instead of a late recall (see Figure 27).

Consequently, we clearly see that the preferences in terms of timing of recall depend on the type of firm. The result that a farsighted firm prefers an early recall could be explained by two elements. First, the earlier the recall, the lower the recall costs to be incurred eventually during the crisis period. Second, as quality is always increasing over time, an early drop when quality has not yet achieved its full potential is easier to recover than when the recall is more distant.

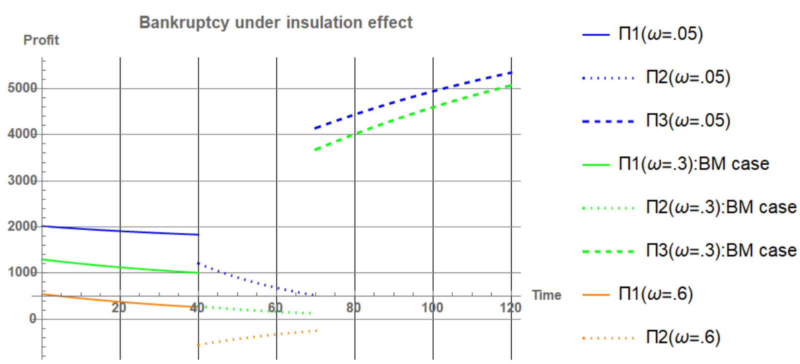

Figure 23: Insulation effect

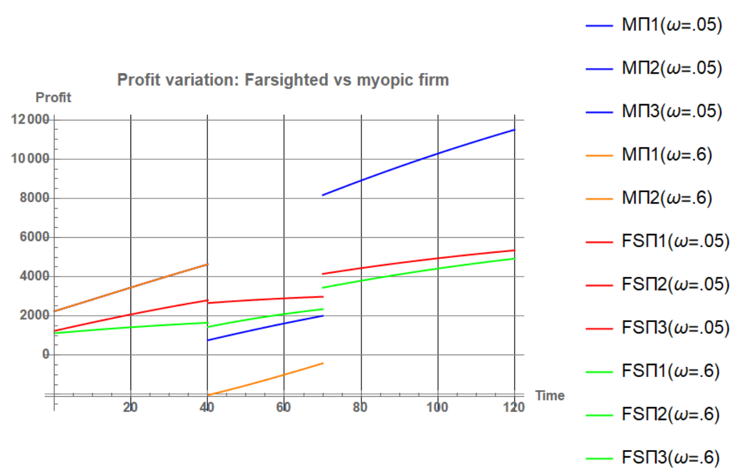

Figure 24: Myopic firm's vs farsighted firm's profit

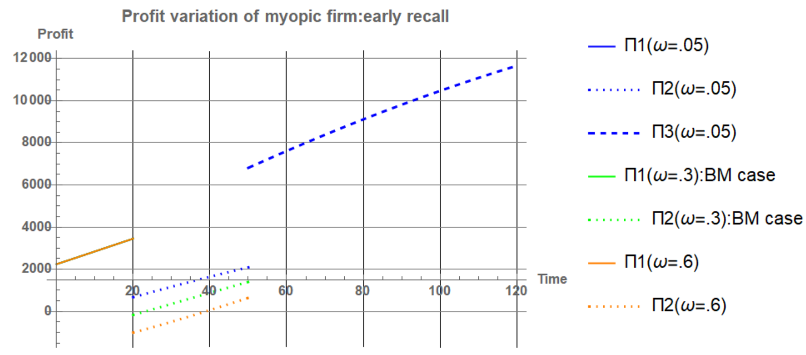

Figure 25: Myopic firm profit with early recall

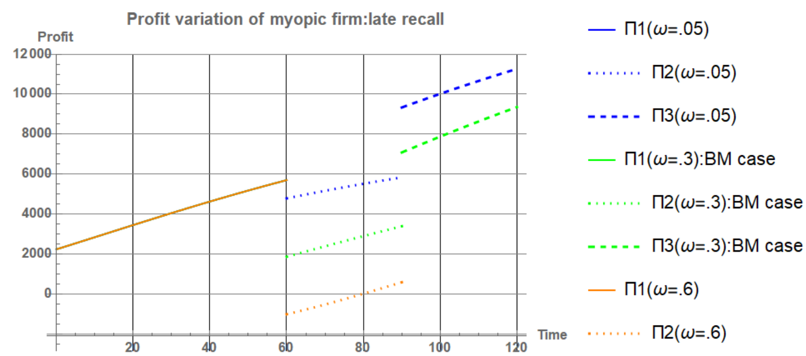

Figure 26: Myopic firm profit with late recall

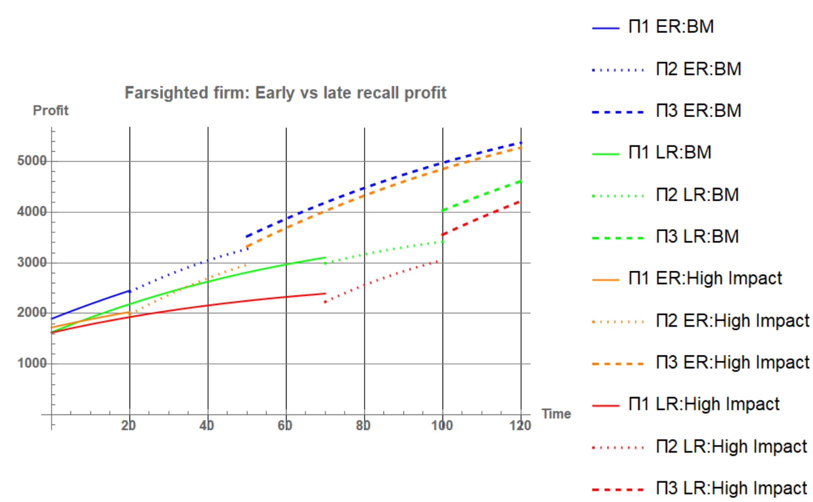

Figure 27: Farsighted firm - early vs late recall profits

6 Conclusion

This paper is motivated by the decision problem of the manufacturers during a product recall. In this paper, we studied precisely how, under the risk of a product recall and anticipation of a subsequent recovery, a manufacturer should channel its efforts into advertising and quality while making pricing decisions. We have also considered how the critical parameters - crisis likelihood, recovery likelihood, crisis impact, and recovery intensity affect the firm's decisions and value functions. Moreover, our analysis of the hazard myopia leads to some interesting insights.

Most of the extant modeling literature focuses on the design quality from a product recall perspective. Inspired by the empirical literature, we modeled the influence of perceived and design quality on the quality evolution of a manufacturer. From the perspective of a product recall, most of the literature mainly focuses on one of the regimes or the pre and post-crisis regimes or only the design quality. We considered a comprehensive analysis of three regimes - pre-crisis, crisis, and post-crisis which makes our model very complex to analyze. Nonetheless, we were able to draw fruitful insights from the combination of analytical and numerical results. Our model analyzes three decisions - price, quality, and advertising. While the empirical literature does an in-depth analysis of the variables in the context of a product recall, modeling literature is scarce and has not discussed the combined effect of three decision variables interacting with each other. Lastly, our methodology adopts a dynamic approach to the solution, which, in the context of a product recall, is superior to a static model as the manufacturer can observe and react to uncertainties in its state dynamics.

From a managerial perspective, our article highlights that a post-crisis recovery is significant for a manufacturer facing a potential product harm crisis. In the absence of recovery, a firm may become bankrupt. Therefore, manufacturers should use proactive recall management strategies to regain consumer trust after a product recall. Examples of such strategies include transparent communication and learning from previous errors. Second, our model is essentially a tool for a manager which generates strategies given the likelihoods and impacts of recall and recovery. Our findings, therefore, suggest that an accurate forecast of a product recall and the impending damage puts a firm in a strategically advantageous position. Third, myopia can be devastating for a firm when a high-impact recall occurs.

There are several possible extensions of this paper. We have considered an optimal control model with one manufacturer. An immediate extension is exploring similar research questions under a differential game by including vertical members of a supply chain or investigating horizontal competition. Second, supplier-manufacturer quality cost-sharing schemes are also of interest in times of product recall. Third, a manufacturer may not have complete information about the hazard rate and recovery rate. Relatively insightful and challenging research can be stimulated by addressing incomplete information under a dynamic game framework.

A Appendix

Proposition 3. *Assuming an interior solution, the optimal advertising, pricing and quality policies are as follows:*

- *Pre-crisis regime:*

$$a_1 = \frac{k_1(2QX_1 + X_2)}{\mu_a}, \tag{A.1a}$$

$$p_1 = \frac{\alpha + \beta\lambda\rho c_r + l_1 X_2 + (2l_1 X_1 + \gamma)Q}{2\beta}, \tag{A.1b}$$

$$q_1 = \frac{m_1(2QX_1 + X_2)}{\theta(\phi - \lambda) + \mu_q}, \tag{A.1c}$$

where $X_j, j = 1, \ldots, 3$ *are the coefficients of the value function* $V_1(Q) = X_1 Q^2 + X_2 Q + X_3$ *and are defined in the Appendix.*

- *Crisis regime:*

$$a_2 = \frac{k_2(2QY_1 + Y_2)}{\mu_a}, \tag{A.1d}$$

$$p_2 = \frac{\alpha + l_2 Y_2 + (2l_2 Y_1 + \gamma)Q}{2\beta}, \tag{A.1e}$$

$$q_2 = \frac{m_2(2QY_1 + Y_2)}{\theta\phi + \mu_q}, \tag{A.1f}$$

where $Y_j, j = 1, \ldots, 3$ are the coefficients of the value function $V_2(Q) = Y_1Q^2 + Y_2Q + Y_3$ and are defined in the Appendix.

- *Post-crisis regime:*

$$a_3 = \frac{k_3(2QZ_1 + Z_2)}{\mu_a}, \tag{A.2a}$$

$$p_3 = \frac{\alpha + l_3Z_2 + (2l_3Z_1 + \gamma)Q}{2\beta}, \tag{A.2b}$$

$$q_3 = \frac{m_3(2QZ_1 + Z_2)}{\theta\phi + \mu_q}, \tag{A.2c}$$

where $Z_j, j = 1, \ldots, 3$ are the coefficients of the value function $V_3(Q) = Z_1Q^2 + Z_2Q + Z_3$ and are defined in the Appendix.

Proof. To show the proposition, we start with solving the problem of the third (post-crisis) regime. Subsequently we solve the second (crisis) regime and, finally, the first (pre-crisis) regime's problem.

To start, we present the problems of the manufacturer in the three periods (the derivation of the problems are elaborated in the main article).

- Post-crisis regime's problem:

$$V_3(Q) = \max_{a_3,p_3,q_3} \int_{t_2}^{\infty} e^{-rt}\Big(p_3\,(t)\,D_3\,(t) - \frac{\mu_a}{2}a_3^2(t) - \frac{\mu_q + \theta\phi}{2}q_3^2(t)\Big)dt \tag{A.3}$$
$$\text{Subject to}$$
$$\dot{Q}(t) = k_3a_3(t) + l_3p_3(t) + m_3q_3(t) - \epsilon_3Q(t), \text{ and } Q(t_2^+) = (1+\psi)Q(t_2^-).$$

- Crisis regime's problem:

$$\begin{aligned}V_2(Q) = \max_{a_2,p_2,q_2} \int_{t_1}^{\infty} & e^{-(r+\chi)t}\Big(D_2(t)p_2(t) - \int_0^{t_1} \rho c_r D(s)ds - \frac{\mu_a}{2}a_2^2(t) - \frac{\mu_q + \theta\phi}{2}q_2^2(t) \\ & + \chi V_3((1+\psi)Q)\Big)dt\end{aligned} \tag{A.4}$$
$$\text{Subject to}$$
$$\dot{Q}(t) = k_2a_2(t) + l_2p_2(t) + m_2q_2(t) - \epsilon_2Q(t), \text{ and } Q(t_1^+) = (1-\omega)Q(t_1^-).$$

- Pre-crisis regime's problem:

$$\begin{aligned}V_1 = \max_{a_1,p_1,q_1} \int_0^{\infty} & e^{-(r+\lambda)t}\Big(D_1(t)p_1(t) - \frac{\mu_a}{2}a_1^2(t) - \frac{\mu_q + \theta(\phi-\lambda)}{2}q_1^2(t) \\ & + \lambda V_2((1-\omega)Q - \rho c_r D_1(t))\Big)dt\end{aligned} \tag{A.5}$$
$$\text{Subject to}$$
$$\dot{Q}(t) = k_1a_1(t) + l_1p_1(t) + m_1q_1(t) - \epsilon_1Q(t), \text{ and } Q(0) = Q_0.$$

From (A.3), the HJB equation in the third regime is:

$$\begin{aligned}rV_3(Q) = \max_{a_3,p_3,q_3} \Big[& p_3\,(\alpha - \beta p_3 + \gamma Q) - \frac{\mu_a}{2}a_3^2 - \frac{\mu_q + \theta\phi}{2}q_3^2 + \\ & \frac{\partial V_3}{\partial Q}\,(k_3a_3 + l_3p_3 + m_3q_3 - \epsilon_3Q)\Big].\end{aligned} \tag{A.6}$$

In order to maximize the value function we take the first-order conditions of the right-hand side of the HJB equation with respect to a_3, p_3 and q_3 and denote the HJB equation of third regime by HJB3.

The first-order conditions yield:

$$\frac{\partial HJB3}{\partial a_3} = -2\mu_a a_3 + k_3 \frac{\partial V_3}{\partial Q} = 0 \tag{A.7}$$

$$\frac{\partial HJB3}{\partial p_3} = \alpha - 2\beta p_3 + \gamma Q + l_3 \frac{\partial V_3}{\partial Q} = 0 \tag{A.8}$$

$$\frac{\partial HJB3}{\partial q_3} = -\frac{\mu_q + \theta\phi}{2} q_3 + m_3 \frac{\partial V_3}{\partial Q} = 0. \tag{A.9}$$

Noting that $V_3(Q) = Z_1 Q^2 + Z_2 Q + Z_3$, $\frac{\partial V_3}{\partial Q} = 2QZ_1 + Z_2$, substituting the value of $\frac{\partial V_3}{\partial Q}$ in (A.7), (A.8) and (A.9) and solving for the decisions a_3, p_3, q_3 respectively, we get:

$$a_3 = \frac{k_3(2QZ_1 + Z_2)}{\mu_a} \tag{A.10}$$

$$p_3 = \frac{\alpha + l_3 Z_2 + (2l_3 Z_1 + \gamma)Q}{2\beta} \tag{A.11}$$

$$q_3 = \frac{m_3(2QZ_1 + Z_2)}{\theta\phi + \mu_q}. \tag{A.12}$$

With the above solution, we proceed to solving the second regime's problem. Form (A.4), the HJB equation of the second period is given by:

$$(r+\chi)V_2(Q) = \max_{a_2,p_2,q_2} \Big[p_2\left(\alpha - \beta p_2 + \gamma Q\right) - \frac{\mu_a}{2} a_2^2 - \frac{\mu_q + \theta\phi}{2} q_2^2 - \rho c_r \int_0^{t_1} D_1(s) ds + \frac{\partial V_2}{\partial Q}(k_2 a_2 + l_2 p_2 + m_2 q_2 - \epsilon_2 Q) + \chi V_3((1+\psi)Q)\Big]. \tag{A.13}$$

We derive the first-order conditions with respect to the decisions of the second period. We denote the HJB equation of second regime by HJB2. The conditions are:

$$\frac{\partial HJB2}{\partial a_2} = -2\mu_a a_2 + k_2 \frac{\partial V_2}{\partial Q} = 0 \tag{A.14}$$

$$\frac{\partial HJB2}{\partial p_2} = \alpha - 2\beta p_2 + \gamma Q + l_2 \frac{\partial V_2}{\partial Q} = 0 \tag{A.15}$$

$$\frac{\partial HJB2}{\partial q_2} = -\frac{\mu_q + \theta\phi}{2} q_2 + m_2 \frac{\partial V_2}{\partial Q} = 0. \tag{A.16}$$

Since t_1 is a deterministic and D_1 is known in the second period, the term $\rho c_r \int_0^{t_1} D_1(s)ds$ is a constant and therefore its derivative will be 0. Now, noting that $V_2(Q) = Y_1 Q^2 + Y_2 Q + Y_3$, $\frac{\partial V_2}{\partial Q} = 2QY_1 + Y_2$, substituting this in equations (A.11), (A.12) and (A.13), and solving for the decision variables, we obtain

$$a_2 = \frac{k_2(2QY_1 + Y_2)}{\mu_a} \tag{A.17}$$

$$p_2 = \frac{\alpha + l_2 Y_2 + (2l_2 Y_1 + \gamma)Q}{2\beta} \tag{A.18}$$

$$q_2 = \frac{m_2(2QY_1 + Y_2)}{\theta\phi + \mu_q}. \tag{A.19}$$

In the first regime, the HJB equation is given by:

$$(r+\lambda)V_1(Q) = \max_{a_1,p_1,q_1} \Big[p_1\left(\alpha - \beta p_1 + \gamma Q\right) - \frac{\mu_a}{2} a_1^2 - \frac{\mu_q + \theta(\phi - \lambda)}{2} q_1^2 + \frac{\partial V_1}{\partial Q}(k_1 a_1 + l_1 p_1 + m_1 q_1 - \epsilon_2 Q) + \lambda(V_2((1-\omega)Q) - c_r \rho D_1)\Big]. \tag{A.20}$$

Proceeding similarly as in the previous cases, and noting that $D_1(t) = \alpha - \beta p_1(t) + \gamma Q(t)$, we derive the first-order conditions with respect to the decision variables p_1, a_1 and, q_1:

$$\frac{\partial HJB1}{\partial a_1} = -2\mu_a a_1 + k_1 \frac{\partial V_1}{\partial Q} = 0 \tag{A.21}$$

$$\frac{\partial HJB1}{\partial p_1} = \alpha - 2\beta p_1 + \gamma Q + l_1 \frac{\partial V_1}{\partial Q} + \beta\lambda\rho c_r = 0 \tag{A.22}$$

$$\frac{\partial HJB1}{\partial q_1} = -\frac{\mu_q + \theta(\phi - \lambda)}{2} q_1 + m_1 \frac{\partial V_1}{\partial Q} = 0. \tag{A.23}$$

Using $V_1(Q) = X_1 Q^2 + X_2 Q + X_3$, we get the pre-crisis regime's decisions

$$a_1 = \frac{k_1(2QX_1 + X_2)}{\mu_a}, \tag{A.24}$$

$$p_1 = \frac{\alpha + \beta\lambda\rho c_r + l_1 X_2 + (2l_1 X_1 + \gamma)Q}{2\beta}, \tag{A.25}$$

$$q_1 = \frac{m_1(2QX_1 + X_2)}{\theta(\phi - \lambda) + \mu_q}. \tag{A.26}$$

□

Proposition 4. *For a hazard-myopic firm the optimal advertising, price and quality policies in regime $i \in \{1, 2, 3\}$ are given by*

$$a_i = \frac{k_1(2Q\Delta_{i1} + \Delta_{i2})}{\mu_a}, \tag{A.27}$$

$$p_i = \frac{\alpha + (2l_1\Delta_{i1} + \gamma)Q}{2\beta}, \tag{A.28}$$

$$q_i = \frac{m_1(2Q\Delta_{i1} + \Delta_{i2})}{\mu_q}, \tag{A.29}$$

where Δ_{i1}, Δ_{i2} and Δ_{i3} are the coefficients of the value function $V_i(Q) = \Delta_{i1}Q^2 + \Delta_{i2}Q + \Delta_{i3}$ and are defined in the Appendix.

Proof. The hazard myopic firm does not see the impending recall or the recovery but it observes the effect of recall and recovery after they occur. Therefore, the problem of a myopic manufacturer is symmetric in every regime. At any time $T \in \{0, t_1, t_2\}$, the manufacturer perceives its problem as follows:

$$V_i(Q) = \max_{a_i, p_i, q_i} \int_T^\infty e^{-rt} \left(p_i(t) D_i(t) - \frac{\mu_a}{2} a_i^2(t) - \frac{\mu_q + \theta\phi}{2} q_i^2(t) \right) dt \tag{A.30}$$

Subject to

$$\dot{Q}(t) = k_i a_i(t) + l_i p_i(t) + m_i q_i(t) - \epsilon_i Q(t), \text{ and } Q(T^+) = (1 + \Omega)Q(T^-)(\text{ and } Q(0) = 0),$$

where

- if $T = 0$, $i = 1, Q(T) = Q(0) = 0$;
- if $T = t_1$, $i = 2, \Omega = -\omega$;
- if $T = t_2$, $i = 3, \Omega = \psi$.

Here t_1 is the random time of recall and t_2 is the random time of recovery.

Therefore, in any regime $i \in \{1, 2, 3\}$, the HJB equation is given by:

$$rV_i(Q) = \max_{a_i, p_i, q_i} \left(p_i(\alpha - \beta p_i + \gamma Q) - \frac{\mu_a}{2} a_i^2 - \frac{\mu_q}{2} q_i^2 + \frac{\partial V_3}{\partial Q}(k_i a_i + l_i p_i + m_i q_i - \epsilon_i Q) \right). \tag{A.31}$$

In order to maximize the value function, we take the first-order conditions of the right-hand side of the HJB equation with respect to a_i, p_i and q_i. The first-order conditions are:

$$\frac{\partial HJBi}{\partial a_i} = -2\mu_a a_i + k_i \frac{\partial V_i}{\partial Q} = 0 \tag{A.32}$$

$$\frac{\partial HJBi}{\partial p_i} = \alpha - 2\beta p_i + \gamma Q + l_i \frac{\partial V_i}{\partial Q} = 0 \tag{A.33}$$

$$\frac{\partial HJBi}{\partial q_i} = -\frac{\mu_q + \theta\phi}{2} q_i + m_i \frac{\partial V_i}{\partial Q} = 0. \tag{A.34}$$

Noting that $V_i(Q) = \Delta_{i1}Q^2 + \Delta_{i2}Q + \Delta_{i3}$, $\frac{\partial V_i}{\partial Q} = 2Q\Delta_{i1} + \Delta_{i2}$,substituting the value of $\frac{\partial V_i}{\partial Q}$ in (A.26), (A.27) and (A.28) and solving for the decisions a_i, p_i, q_i respectively, we get

$$a_i = \frac{k_1(2Q\Delta_{i1} + \Delta_{i2})}{\mu_a} \tag{A.35}$$

$$p_i = \frac{\alpha + (2l_1\Delta_{i1} + \gamma)Q}{2\beta} \tag{A.36}$$

$$q_i = \frac{m_1(2Q\Delta_{i1} + \Delta_{i2})}{\mu_q}. \tag{A.37}$$

□

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