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A survey of dynamic models of product quality

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Abstract : We review dynamic quality models both in single-agent setup and in a competitive framework. Our objectives are: (1) to give the reader a vantage point on the state of the art in this area, (2) to identify the boundaries between the different concepts of quality to help build a bridge between the various communities interested in the management of quality, and (3) to sketch out a research agenda for the area. The paper should not only be of interest to active researchers in the field, but also to a broader community of scholars and practitioners working in operations management, marketing, industrial engineering and operations research, who are interested in quality dynamics.

Keywords: Quality Management, optimal-control models, differential games, survey

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1 Introduction

During the three last decades or so, a large literature has developed on product quality management, with one stream using a dynamic model to reflect the idea that firms can improve the quality of their products over time. This paper surveys this dynamic literature both in a one-firm framework and in a competitive setup.

We all have probably witnessed two friends discussing a product quality or brand's quality and ending in a deadlock. One likely reason for this outcome is that each party had her own personal definition of quality. Such differences in opinion also occasionally involve consumers and firms. For instance, a new product can be a true marvel from an engineering and production point of view, but be snubbed by consumers because it does not suit their tastes, and therefore be labeled a poor-quality product. Then, one can ask the following simple question: can quality be assessed through a reliable and valid measurement scale? In a seminal paper, Garvin (1988) suggested the following eight dimensions to measure this construct:

1. **Performance** corresponds to a product's primary operating characteristics and involves measurable attributes.
2. **Features** are additional characteristics and options that complement the basic functions, and make the product more appealing and the service more useful.
3. **Reliability** is the likelihood that a product does not fail within a specific time period.
4. **Conformance** is the capability of a product or a service to meet the specified standards.
5. **Durability** measures the length of a product's life and may be defined as the amount of use a consumer gets from a product before it breaks down and replacement is preferable to repair.
6. **Serviceability** refers to the consumer's ease of obtaining repair service, the responsiveness of service personnel, and the speed at which the product can be put into service after it breaks down.
7. **Aesthetics** indicates how the product looks, feels, sounds, tastes, or smells. This is a matter of personal judgement and a reflection of individual preference.
8. **Perceived quality** is an assessment of the quality of a good or service based on indirect measures.

Clearly, Garvin's dimensions integrate both the firms' and consumers' points of view. Whereas the first six dimensions can be considered, at least to some extent, as objectively measurable, the last two are more judgement-based. From a managerial point of view, a total quality management (TQM) approach means that all dimensions have been addressed by the firm, in one or another way.

Two general observations can be made from the outset. First, we are not aware of a single contribution in the surveyed literature that built a model integrating all of Garvin's dimensions. Since characterizing optimal strategies in a dynamic model, especially in a competitive environment, is never easy, it is understandable to aim for a parsimonious model that focuses on the fundamentals. Schematically, the reduction of the dimensionality has been done in two ways: either by selecting few of the eight dimensions in the analysis, or by aggregating them into few factors, e.g., objective quality and perceived quality. To illustrate, a series of contributions only retained Garvin's last dimension. Here, what matters is how the consumer perceives quality, either before, or after consuming the product. Another example is the concept of *design quality*, which is popular in the literature, and defined by Fine (1986) as "..features, styling, and other product attributes that enhance the fitness for use or utility for the consumers..." From this, one can conclude that any investment made to increase the consumer's utility and, in turn, sales, qualifies to fall under the concept of design quality, which does not (exclusively) correspond to any of the dimensions in Garvin (1988). Kouvelis and Mukhopadhyay (1995a) further broaden the scope of design quality by letting it be "...a long way than the Garvin's quality dimensions; beyond including all the Garvin's dimensions, it aims at increasing the product desirability in the market and then the firms' profitability..."

Second, the terminology used in the literature is far from being unique. For instance, the literature, at least in some instances, interchangeably used *design quality* and *quality improvement* to describe a state reached by a product through innovation, additional features, and options. This has made the distinction between radical innovation (e.g., a first-generation mobile phone versus a smart phone) and incremental innovation (adding gadgets to a cell phone), become less clear.

The objective of this survey is threefold. First, to give the reader a vantage point on the state of the art in this area. Second, to identify the boundaries between the different concepts of quality to help building a bridge between the various communities interested in the management of quality. And finally, to set what we believe should be a research agenda in this area. In particular, we discuss some possible extensions to Garvin's scale to account for some new trends and practices in the management of quality. We mention from the outset that this survey aims to be useful not only to active researchers in the field but also to a larger community of scholars and practitioners working in operations management, marketing, industrial engineering, and operations research, who are interested in quality dynamics.

The survey includes 103 contributions,[1] whose sources are listed in Table 1, which were selected according to a single criterion: quality must vary over time. Accordingly, static models were disregarded, as were multiperiod models where the quality decision is made only once. In the retained set, quality can be a control (or decision) variable whose intertemporal values result from the optimization of a certain objective (typically profit), or a state variable whose evolution over time depends on some control variables, e.g., investment in quality improvement or advertising. The surveyed papers are described in the Appendix in terms of the type strategic interaction (horizontal (oligopoly), vertical (supply chain), or no interaction (single-agent problem), the model (deterministic or stochastic, finite or infinite planning horizon), the decision variables (controls in optimization problems and strategies in a game setup), the dynamics, the functional form of the cost in quality investment, and the sales function.

Table 1: Number of reviewed papers by source

Journal	Number of reviewed papers
European Journal of Operational Research	26
Annals of Operations Research	5
Management Science	5
Journal of Operations Research Society	4
International Journal of Production Economics	4
Journal of Optimization Theory and Applications	4
Computers & Industrial Engineering	3
Decision Sciences	3
Omega	3
Operations Research Letters	3
Economics Letters	2
IIE Transactions	2
International Game Theory Review	2
Journal of Operations Management	2
Marketing Letters	2
Operations Research	2
Other Journals	20
Books and Book Chapters	11

The rest of the paper is organized as follows: Sections 2 and 3 present the parts of the literature that have considered quality a control variable, or a state variable, respectively. Section 4 reports on the impact of quality on other variables, and Section 5 discusses the relationships between quality and sales. Section 6 summarizes how quality affects production cost, and Section 7 concludes.

[1] In the Appendix, 71 papers are listed, the other references have been useful for providing some background, mentioning an extension, reporting on related issues, etc.

2 Quality as a decision variable

In this section, we report on the contributions that considered quality a control variable or a strategy that influences either sales, state dynamics, or both. We will come back to this impact itself in the next sections.

Denote time by t and let T be the planning horizon, which can be finite or infinite. In the literature, various variables have been used to define quality, which can be categorized into four categories: conformance quality $q^{\mathcal{C}}(t)$, design quality $q^{\mathcal{D}}(t)$, quality improvement $q^{\mathcal{I}}(t)$, or quality experience $q^{\mathcal{E}}(t)$. We note from the outset that only conformance quality corresponds, in a strict sense, to one of Garvin's dimensions. The other measures were essentially obtained by combining several of these dimensions into one.

In Fine (1986), investing in conformance quality helps in learning to produce defect-free goods. In Chand et al. (1996), $q^{\mathcal{C}}(t)$ corresponds to the effort spent on human resources and equipment to achieve the conformance quality goal. De Giovanni (2020) considers appraisal efforts, which include inspection, testing, and supervision related to these activities, and prevention efforts to account for quality engineering, training, and related supervision costs. These efforts make it possible to reach a desired conformance level and enable firms to establish and maintain high-level system performance. De Giovanni and Tramontana (2016) propose a variation on traditional conformance quality efforts, whose amplitude depends on the gravity of the failures to be recovered. By using a Failure Mode and Effect Analysis (FMEA), nonconformities leading to catastrophic events (e.g., human deaths and health issues) require sizeable investments in quality to quickly recover performance. However, a minor failure can be easily recovered via minimal investments in quality, complemented by an effective marketing campaign. Li and Rajagopalan (1998) link conformance quality to quality improvements, quality assurance, and quality for external failures. Quality improvements refer to the percentage of conforming products made by a production process. Quality assurance includes costs for inspections and internal failures as well as investments in external failures that are modeled as a function of the failures experienced by consumers (De Giovanni, 2020).

We synthesize the above various definitions as follows:

Conformance quality is the firm's capability to make defect-free goods. This capability is acquired by investing in appraisal, inspection, prevention, failure recovery, quality assurance, and learning.

Remark 1. There is a literature dealing with product recall: events triggered by defective goods. The contributions in this area typically looked at quality, pricing, and advertising decisions before and after the recall; see, e.g., Rubel et al. (2011), Rubel (2018), Lu and Navas (2021), and Mukherjee and Chauhan (2021).

We mentioned in the introduction the contributions of Fine (1986) and Kouvelis and Mukhopadhyay (1995a) in defining design quality, a variable that plays a direct or indirect role in consumer's utility and/or sales. Other authors linked design quality to:

- Attributes that consumers always prefer more to less and that can be easily searched for (Teng and Thompson, 1996);
- R&D efforts for upgraded material usage, production process methods, product features, and tighter controls (Mukhopadhyay and Kouvelis, 1997);
- Overall effect of multiple product attributes (Mukhopadhyay and Setaputra, 2007).
- Efforts to better understand consumers' needs and to design products accordingly (De Giovanni, 2021a).

Based on the above, we propose the following definition:

Design quality is the firm's capability to engineer and make quality goods, in the sense of all of Garvin's dimensions, that match consumers' needs and maximize the firm's outcome. This capability is acquired over time by investing in product innovation, technology, methods, and processes.

Although there is (by the choice of survey area) a consensus in the literature that the quality of an existing product can be changed over time through costly investments, the authors took different routes to operationalize this. For instance, Martín-Herrán et al. (2012) refer to changes in the performance, functionality, safety, and packaging of a product as *quality enhancement*. De Giovanni (2011, 2013) refers to investments to have a superior good that strengthens the brand's value as *quality improvements*. For Liu et al. (2015b), it is the costly efforts to improve product performance. Regardless of the terminology used, these papers refer mainly to Garvin's first dimension (Performance) in their description of quality. Liu et al. (2016) refer to quality in terms of product features (Garvin's dimension 2), while Lu and Navas (2021) encompass Garvin's dimensions 1 and 6 (Performance and Serviceability) in their modeling of quality. Vörös (2019) sees *quality performance* as the result of primary, or core, attributes, and some secondary attributes, which encompass Garvin's dimensions 1 and 2. Narasimhan and Ghosh (1994) adopt a things-gone-wrong measure with reference to customer satisfaction and product quality. That is, a global measure of quality that includes reliability, conformance, and durability (Garvin's dimensions 3, 4, and 5). Finally, Wang and Li (2012) adopt a negative gauge, *quality deterioration*, i.e., the rate at which quality characteristics and features naturally decrease over time (Garvin's dimension 5).

In another series of papers, quality improvement is taken as synonymous to *product innovation*, which must, in our opinion, be understood in an incremental sense; otherwise, if the innovation is radical, then we would be dealing with a situation where the firm sells different products (or versions of a product) over time. For instance, Bayus (1995) considers that product/quality improvement is due to product innovation strategies, whereas Ni and Zhao (2021) state that product quality is achieved through product innovation. For others, product innovation consists of investments made to increase the quality of a good or to avoid obsolescence (Chenavaz, 2011 and 2012; Lambertini and Orsini, 2015; Chenavaz and Jasimuddin, 2017; Li, 2021; Zhao and Ni, 2021). Also, the objective of such investments could be to increase product differentiation to gain a competitive advantage (Li, 2017; Lambertini and Mantovani, 2009; Bao and Ma, 2017), to reduce emissions and environmental impact (Wang et al., 2019), to improve green product quality (Mukherjee and Carvalho (2021), or to reach a quality certification level (Li and Ni, 2018).

One definition that captures the different points of view is as follows:

Quality improvement is the firm's capability to raise the quality of existing goods. This capability is acquired over time by investing in, e.g., R&D, (incremental) product innovation, upgraded materials, new production methods and safe procedures, tighter controls, packaging, capacity to adjust to consumer changes, lower emissions and environmental impact, and satisfaction of quality standards and regulations.

Finally, we aggregate under *quality experience* all efforts made by the firm to better serve the consumer. Li and Rajagopalan (1998) call such efforts *process improvements*, while De Giovanni (2021b) refers to them as *quality investments* needed to better understand consumer needs and utility, and then make products that match consumer expectations. Nair and Narasimhan (2006) define *quality efforts* as the investments required to deliver goods that are better than their competitors.

To make the above investments successful, the firm needs to understand how the consumer judges quality. One approach consists in assuming that the consumer benchmarks product quality against a *reference quality*, which is learned by a weighted average of past observed quality values (Kopalle and Winer, 1996). (This dynamic process is similar to the one used to define reference price.) When the observed (or estimated) quality is equal to the reference quality, then it is termed *perfect quality* in Caulkins et al. (2017). If the actual quality is higher (lower) than the reference quality, then the consumer obtains a gain (loss) if she buys the product. A gain and a loss of the same magnitude need not be weighted in the same manner (Gavious and Lowengart, 2012). We note that the moment at which a consumer can assess the quality of a product, whatever metric she has in mind, varies with the type of product. For instance, the quality of so-called experience goods, e.g., perfume or meal in a restaurant, can only be determined after consumption (Kotowitz and Mathewson, 1979). The quality of a credence good may not be precisely assessed even after consumption. Typical examples include professional services, e.g., health and legal services. Here, the firm invests in quality to get recognition and certifications from inspections by authorities and certification bodies (Hirschmann, 2014).

Although the definitions of design quality and quality improvements are large, they do not include the idea of both parties (firm and consumer) "learning" quality. We propose the following definition to complete the picture:

Quality experience is the firm's capacity to understand the consumer's needs and ensure a good's high performance to achieve customer satisfaction, contribute to the firm's reputation, and improve the consumer's journey. This capability is acquired over time by investing in attributes, research, consumer satisfaction, features, options, and performance.

3 Quality as a state variable

A large number of contributions considered quality as a state variable, implying that the quality of the product currently offered to consumers is the result of a series of decisions made over time. For instance, a low percentage of defects at time t is due to all the past investments in conformance quality. The literature has given different names to this state variable, which we categorize under the following headings: conformance quality, objective quality, and others.

As any typology, ours is not unique. We wanted to have groups that are inclusive, i.e., all papers must find a home, and to stay close to Garvin's dimensions. Unfortunately, there is no way of having categories that are mutually exclusive because, e.g., perceived quality mixes both objective and subjective dimensions of quality. Having only three categories, allows us to capture the essence of the literature in a parsimonious way.

3.1 Conformance quality

Conformance quality, denoted by $Q^{\mathcal{C}}(t) \in [0,1]$, represents the percentage of defect-free products that a firm is able to make. The lower bound means that all products are defective, while $Q^{\mathcal{C}}(t) = 1$ corresponds to the case where all manufactured products meet the specifications. Therefore, $1 - Q^{\mathcal{C}}(t)$ represents the fraction of defective goods made and sold in the market.

Chand et al. (1996) and El Ouardighi et al. (2008, 2013, 2016, 2018) describe the evolution of the percentage of defect-free products by the following differential equation:

$$\dot{Q}^{\mathcal{C}}(t) = q^{\mathcal{C}}(t)\left(1 - Q^{\mathcal{C}}(t)\right), \quad Q^{\mathcal{C}}(0) = Q_0^{\mathcal{C}}, \tag{1}$$

where $q^{\mathcal{C}}(t)$ is the investment in conformance quality, and $Q_0^{\mathcal{C}} \in (0,1]$ is the initial conformance quality. The above dynamics show that the improvement in conformance quality becomes slower as the firm approaches its target of zero defective items. El Ouardighi and Pasin (2006) retain (1) in a duopoly where firms interact in the market, but not in their management of quality, that is, each firm's rate of defect-free products is independent of other players' investments in conformance quality. This could be otherwise if the firms learned from each other, be it voluntarily (by sharing expertise) or involuntary (by industrial espionage or reverse engineering). El Ouardighi and Kogan (2013) extend (1) to a supply chain where both the manufacturer (firm M) and the supplier (firm S) contribute to the conformance quality dynamics by investing in $q_M^{\mathcal{C}}(t)$ and $q_S^{\mathcal{C}}(t)$, respectively. The dynamics are then given by

$$\dot{Q}^{\mathcal{C}}(t) = \left(q_M^{\mathcal{C}}(t) + q_S^{\mathcal{C}}(t)\right)\left(1 - Q^{\mathcal{C}}(t)\right), \quad Q^{\mathcal{C}}(0) = Q_0^{\mathcal{C}}.$$

In (1), the investment can be seen as a reaction to reduce the percentage of defective items. De Giovanni (2020) builds on the model of Chand et al. (1996) by incorporating an additional proactive term for a better quality management. A proactive approach considers the systematic variability within a production process. In fact, the quality varies naturally with changes in circumstances and resources (Crosby, 1979). For example, a worker who gets tired by the end of the day has a lower level of attention, which may increase the number of defects. Similarly, some maintenance has to be carried out on a machine after a certain number of hours of operation; otherwise, the number of defects increases. By investing in appraisal and prevention, the firm mitigates the decay in quality. The evolution of conformance quality can then be modeled as follows:

$$\dot{Q}^{\mathcal{C}}(t) = \eta_{q^{\mathcal{A}}} \cdot q^{\mathcal{A}}(t) + \eta_{q^{\mathcal{C}}} \cdot q^{\mathcal{C}}(t)\left[1 - Q^{\mathcal{C}}(t)\right] - \varepsilon Q^{\mathcal{C}}(t), \quad Q^{\mathcal{C}}(0) = Q_0^{\mathcal{C}}, \tag{2}$$

where $q^{\mathcal{A}}(t)$ is the investment in appraisal and prevention; $\eta_{q^{\mathcal{A}}}$ and $\eta_{q^{\mathcal{C}}}$ are the appraisal and conformance effectiveness parameters, respectively; and ε is the decay in conformance quality if no investment is made in appraisal and conformance quality.

Using a discrete-time model, De Giovanni and Tramontana (2016) propose the following dynamics of conformance quality:

$$Q^{\mathcal{C}}(t+1) = Q^{\mathcal{C}}(t) + s_Q \frac{\partial \Pi_M}{\partial Q^{\mathcal{C}}(t)}, \tag{3}$$

where Π_M is the manufacturer's profit and s_Q is a positive parameter. Therefore, the manufacturer invests in conformance quality according to its marginal contribution to profit, scaled by s_Q, which is interpreted as the speed of adjustment.

In the above references, the assumption is that the production system is a single-stage one. Kogan and Raz (2002) is the only paper that considers a system consisting of S stages, and I possible inspection activities that can be carried out in each stage. A defect that occurs at any stage of the production process is either detected and removed by inspection at that stage or escapes the inspection and propagates to the next stage, which induces an additional cost. In other words, the state variable is the cumulative change in the number of undetected defects in stage s at time t, and its variation is given by the difference between the defects introduced in previous stages and those successfully removed.

Making high-quality goods, in a defect-free sense, is a learning process. Using a discrete-time framework, Tapiero (1987) models experience in manufacturing, which can be considered a proxy of conformance quality, as a state variable whose evolution depends on production (learning-by-doing effect) and on (random) quality inspections. Further, Fine (1986) proposes the following dynamics:

$$\dot{Q}^{\mathcal{C}}(t) = \eta_S \cdot S(t) + \eta_{q^{\mathcal{C}}} \cdot q^{\mathcal{C}}(t) + q^{\mathcal{C}}(t) S(t), \tag{4}$$

where $S(t)$ represents sales at time t, and η_S is a positive scaling parameter. The dynamics in (4) state that conformance quality benefits from sales (learning-by-doing effect) and investments in making defect-free goods, with an additional interaction term that amplifies these two benefits. Foster and Adam (1996) use the same approach undertaken by Fine (1986) while letting the end of the planning horizon be a function of the speed of quality improvements. Kogan and El Ouardighi (2019) also allows for learning conformance quality (not called as such, but it fits our definition). Here, each of the competitors in a Bertrand game learns by doing, that is, from experience in production, and also by investing in quality learning (called induced learning).

3.2 Objective quality

In this category, we grouped together all the state dynamics in the literature whose right-hand side only involves efforts related to quality improvement. The general form of the state equation is as follows:

$$\dot{Q}^{o}(t) = f\left(q(t), Q^{o}(t)\right), \quad Q^{o}(0) = Q_0^{o}, \tag{5}$$

where $Q^o(t)$ is the (objective) product quality at t, $q(t)$ is the investment made to increase this quality, and Q_0^o is the initial value of the product quality. We reiterate that the labels given to $Q^o(t)$ and $q(t)$ are ours. To illustrate the variety of definitions used, we note that $q(t)$ has also been referred to as quality improvement, quality, product innovation, product innovation to make green goods, product innovation to achieve quality authorization, quality features, quality enhancer, product improvement, and quality development. Similarly, $Q^o(t)$ has been termed quality, quality design, quality improvement, quality certification, and green quality. (Additional details are provided in the Appendix.)

In its simplest expression, (5) takes the form of a linear differential equation, i.e.,

$$\dot{Q}^o(t) = \eta q(t) - \delta Q^o(t), \quad Q^o(0) = Q_0^o, \tag{6}$$

where η is a parameter measuring the efficiency of the investment q, and δ is a decay rate capturing the obsolescence in quality. The above specification has been very popular in the literature and adopted in, e.g., Li and Ni (2016, 2018), Pan and Li (2016), Lambertini et al. (2017), Zhong and Zhang (2018), Wang et al. (2019), Li (2021), De Giovanni (2021c), and Ni and Zhao (2021). In a few papers, $Q^o(t)$ measures design quality as in, e.g., Kouvelis and Mukhopadhyay (1995b) and Cohen et al. (1996). In these last two references $\delta = 0$, that is, there is no decay in design quality. See also Mukhopadhyay and Setaputra (2007), Vörös (2019), and De Giovanni (2021a). In Cohen et al. (1996), where the focus is on new product development, there is a date at which the investment in design quality stops and an investment in quality process starts. It ends when the product is launched.

A series of modifications to (6) have been proposed. Vörös (2019) considers that the product quality evolution depends on some activities that require investments and have a long-term impact, e.g., hiring researchers, and others that do not have cumulative effects. A special case of (6) is when $\eta = 0$, that is, the quality naturally deteriorates over time, which is the case for food. For instance, Wang and Li (2012) use $\dot{Q}^o(t) = -\delta (Q^o(t))^\chi$, where χ captures the chemical reactions involved in quality deterioration. De Giovanni (2021d) added a given reference quality q_R to (6), that is,

$$\dot{Q}^o(t) = q(t) + \psi (q(t) - q_R) - \delta Q^o(t), \tag{7}$$

where ψ is a positive parameter. Reference quality effect is also studied in Chenavaz (2017).

To account for marginal decreasing returns in quality investment (innovation), Chenavaz (2011) retains the following dynamics:

$$\dot{Q}^o(t) = \eta \ln q(t) - \delta Q^o(t), \quad Q^o(0) = Q_0^o,$$

with $\delta \in [-1, 1]$. A positive (negative) value of δ corresponds to a deterioration (improvement) in the quality process. An interaction term between the control and state variables has also been considered. For instance, Lambertini (2015) proposes

$$\dot{Q}^o(t) = q(t) Q^o(t) - \delta Q^o(t), \quad Q^o(0) = Q_0^o,$$

which means that the higher the current product quality, the higher the impact of the investment. On the contrary, Chenavaz et al. (2020) assume that

$$\dot{Q}^o(t) = \sqrt{q(t)}\, (Q^o)^{-\alpha}(t) - \delta Q^o(t), \quad Q^o(0) = Q_0^o,$$

where the positive parameter α reflects the idea that it is harder to further improve quality, if it is already high.

Mukhopadhyay and Kouvelis (1997) extend the approach used in Kouvelis and Mukhopadhyay (1995b) to a duopoly. Here, each firm invests $q_j(t), j = 1, 2$ to make a superior quality good through more precise processes, expensive controls, product features, and new materials. The dynamics of each firm depends only on its own investment, while both firms' quality design stocks affect both firms' demands. A supply chain is considered in El Ouardighi and Kogan (2013), El Ouardighi (2014), and El Ouardighi and Shniderman (2019), with the dynamics of (design) quality given by

$$\dot{Q}^o(t) = \eta\left(q_M(t) + q_S(t)\right) - \delta Q^o\left(t\right), \quad Q^o\left(0\right) = Q_0^o,$$

where $q_M(t)$ and $q_S(t)$ represent the investment made by the manufacturer (firm M) and a supplier (firm S), respectively. Note that product quality here is a public good to which both partners contribute, and one expects the investments to be higher when both players cooperate than when they do not (and both free ride). The same form is used in Lambertini (2018), where, however, the two players are divisions within a same firm. El Ouardighi and Kim (2010) extend the model to a supply chain with one supplier and two competing manufacturers. Each manufacturer invests in its own design quality, and the supplier contributes to both manufacturers' design quality stocks.

3.3 Other definitions

In this section, the product quality is the result either of past actions undertaken to improve its usability (performance, durability, etc.), or some signals to which the consumer has been exposed (in particular, price and advertising). Schematically, the evolution of quality can be represented by the following differential equation:

$$\dot{Q}(t) = f\left(Q\left(t\right), q(t), p\left(t\right), A\left(t\right), X\left(t\right)\right), \quad Q\left(0\right) = Q_0, \tag{8}$$

where $p\left(t\right)$ is the price, $A\left(t\right)$ is advertising, $X\left(t\right)$ is a vector of any other relevant variables, and, as before, $q(t)$ is the investment in quality improvement.

In a series of papers, the state variable is defined as *perceived quality*, which we denote by $Q^P\left(t\right)$. For instance, in Fruchter (2009), the firm uses price and advertising to signal its product's (perceived or overall) quality. In this case, (8) takes the form

$$\dot{Q}^P(t) = \alpha p(t) + \beta A\left(t\right) - \delta Q^P\left(t\right), \quad Q^P\left(0\right) = Q_0^P,$$

where $\alpha > 0$ and $\beta > 0$ measure the marginal impact on quality of price and advertising, respectively. Here, the higher the price, the higher the perceived quality. In Martín-Herrán et al. (2012), the perceived (brand) quality evolves as follows:

$$\dot{Q}^P(t) = q(t) + \beta\left(p(t) - p_R(t)\right) - \delta Q^P\left(t\right), \quad Q^P\left(0\right) = Q_0^P, \tag{9}$$

where $p_R(t)$ is the reference price, a state variable whose evolution is governed by the following differential equation:

$$\dot{p}_R(t) = \gamma(p(t) - p_R(t)), \quad p_R(0) = p_R^0,$$

where γ is the speed of adjustment of the reference price. In this model, the perceived quality depends on the investment in quality and on the price signal. If the price is higher (lower) than the perceived price, then consumer increases (decreases) her assessment of the product quality. Hirschmann (2014) uses a stochastic differential equation to model the evolution of perceived quality, with the publicly observable quality being referred to as reputation. Narasimhan et al. (1993, 1996) and Narasimhan and Mendez (2001) model, among other things, the process by which perceived quality (and quality reputation) is formed, with the objective of assessing its effect on sales and, ultimately, on the firm's profitability.

Reference quality is another state variable that has been used in the literature. Like reference price, reference quality is the result of a learning process. The consumer forms a belief about quality, which could be called perceived quality, and adjusts this belief once a new observation of quality is available; see, e.g., Kopalle and Winer (1996), Gavious and Lowengart (2012), Liu et al. (2016), Xue et al.

(2017), and Li et al. (2020). To illustrate, in Kopalle and Winer (1996), the dynamics of reference quality are as follows:

$$\dot{Q}^R(t) = \sigma\left(q(t) - Q^R(t)\right) + \phi\left(p(t) - p_R(t)\right), \quad Q^R(0) = Q_0^R, \tag{10}$$

where σ and ϕ are the speed of adjustment of quality and price, respectively. In Gavious and Lowengart (2012) and Liu et al. (2016), the reference quality is independent of the price, that is, $\phi = 0$. In Xue et al. (2017), Li (2021), and Li et al. (2020), ϕ is also equal to zero, with the observed quality being a state (not control) variable that evolves according to (6). Therefore, in these papers, the model includes two state variables.

The idea of the consumer learning product quality was already in Kotowitz and Mathewson (1979). These authors assumed that quality has two aspects: an instantaneous verifiable quality and a quality variable that requires experience to be assessed. They modeled quality experience as in (10) with $\phi = 0$. The model was also expanded to account for advertising as follows:

$$\dot{Q}^E(t) = \sigma\left(q(t) - Q^E(t)\right) + F\left(q(t) - Q^E(t), A\right), \quad Q^E(0) = Q_0^E, \tag{11}$$

where Q^E is quality experience and A is the persuasive advertising effort. Different functional forms of $F(\cdot)$ are discussed in the paper.

Remark 2. Quality experience has been used to describe the learning of quality by the consumer, as in Kotowitz and Mathewson (1979), and to describe the firm's learning of how to produce high-quality goods, as in, e.g., Vörös (2006), Li and Rajagopalan (1998), Carrillo and Gaimon (2000), Dawin et al. (2015) and De Giovanni (2021b). In this second sense, the drivers of quality experience (or knowledge) are investments in product performance, in processes, in R&D, in sales, etc. In Gaimon (1988a,b), acquiring experience is done by making adjustments to the production capacity (e.g., investing in new technology) and/or exploiting the salvage value linked to the existing capacity.

4 Impact of quality on other state variables

A series of contributions let quality, as a state or control variable, influence the evolution of an other state variable. In particular, some authors added quality to the goodwill dynamics in Nerlove and Arrow (1962). In its original form, the goodwill dynamics have the following form:

$$\dot{G}(t) = \gamma A(t) - \delta G(t), \quad G(0) = G^0,$$

where $G(t)$ is the firm's (or brand's) goodwill at time t, $A(t)$ represents the advertising effort, and γ and δ are positive parameters capturing the impact of advertising and the decay in goodwill due to consumer's forgetting effects.

De Giovanni (2011) extends the above dynamics to

$$\dot{G}(t) = \gamma A(t) + \epsilon q(t) - \delta G(t), \quad G(0) = G^0, \tag{12}$$

where ϵ is a positive parameter. Later on, De Giovanni (2013) proposed

$$\dot{G}(t) = \gamma A(t) + \epsilon q(t)\sqrt{G(t)} - \delta G(t), \quad G(0) = G^0,$$

that is, the larger the goodwill, the larger the impact of quality, with the effect being subject to marginal decreasing returns, captured by the concave function $\sqrt{G(t)}$. Cesaretto et al. (2021) adopt the following dynamics:

$$\dot{G}(t) = \gamma A(t) + \epsilon q(t)\sqrt{G(t)} - \sigma\left(q(t)\right)^2 - \delta G(t), \quad G(0) = G^0.$$

Again, the marginal decreasing returns in quality effort are captured by the concavity of $\dot{G}(t)$ with respect to $q(t)$.

Reddy et al. (2016) assume that the decay rate is a decreasing function of quality investment, that is,

$$\dot{G}(t) = \gamma A(t) - \delta(q(t)) G(t), \quad G(0) = G^0,$$

where

$$\delta(q(t)) = \frac{2\delta}{1 + e^{k(q-\bar{q})}},$$

with k and $\bar{q}$ being positive parameters. The parameter $\bar{q}$ is interpreted as the average or expected quality. If the product quality is constantly equal to the expected quality, then the decay rate becomes constant and given by $\delta(q(t)) = \delta$. This means that the original Nerlove-Arrow model is a particular instance of the model proposed in Reddy et al. (2016). Further, the function $\delta(q(t))$ is S-shaped and is concave for $q \geq \bar{q}$ and convex for $q \leq \bar{q}$. This asymmetry in curvature, depending on whether or not quality exceeds expectation, is richer than having a monotone first derivative.

Buratto et al. (2019) replaced advertising by the product price in (12), that is,

$$\dot{G}(t) = \vartheta p(t) + \epsilon q(t) - \delta G(t), \quad G(0) = G^0,$$

where ϑ is a positive parameter. Here, the higher the price, the higher the firm's goodwill. Ni and Li (2019) conduct a general analysis without specifying the functional form of the dynamics and consider

$$\dot{G}(t) = f(A(t), q(t), G(t)), \quad G(0) = G^0,$$

with f being an increasing function in its first two arguments.

In the above papers, it is the investment in quality improvement that enters the goodwill dynamics. In the next two contributions, it is quality as a state variable that does. De Giovanni (2020) proposes

$$\dot{G}(t) = \gamma A(t) - \kappa(1 - Q(t)) - \delta G(t), \quad G(0) = G^0,$$

that is, the goodwill evolution depends on quality conformance measured, as usual, by $1 - Q(t)$. As κ is positive, then a firm with a lower number of defects enjoys a higher reputation. Lu and Navas (2021) assume that the impact of advertising on goodwill interacts with the product quality, that is,

$$\dot{G}(t) = \gamma A(t) \sqrt{Q(t)} - \delta G(t), \quad G(0) = G^0.$$

The square root is used to capture the marginal decreasing returns of quality on goodwill.

Nair and Narasimhan (2006) consider a duopoly and assume that each firm's goodwill dynamics depend positively on its advertising and quality efforts and negatively on its rival's advertising and quality investments. Interestingly, whereas typically the literature starts by looking at a single-firm setup using an optimal-control model before extending the formalism to an oligopoly using a differential game, here, the first paper that integrated quality to the Nerlove-Arrow model did it in a competitive setting.

Another way to capture dynamic competition in a duopoly is by adopting a Lanchaster market-share model defined by

$$\dot{x}(t) = f(A_1(t))(1 - x(t)) - f(A_2(t)) x(t), \quad x(0) = x_0, \tag{13}$$

where $x(t) \in (0,1)$ is the market share of firm 1, and hence the rival's market is given by $1 - x(t)$. In this model, each firm uses advertising to attract the competitor's consumers. Ringbeck (1985) includes quality in the above model as follows:

$$\dot{x}(t) = A(t)(1 - x(t)) - \delta(A(t), q(t)) x(t), \quad x(0) = x_0. \tag{14}$$

In this variant, firm 1 uses advertising to increase its market share among non-consumers (untapped market) and both advertising and quality to keep its own customers.

El Ouardighi and Pasin (2006) include conformance quality in the Lanchaster model in which the defective items are a driver of the market share. Specifically, firm 1's market share evolves according to the dynamics:

$$\dot{x}(t) = w_1 G_1(t)\left(1 - Q_2(t)\right)\left(1 - x(t)\right) - w_2 G_2(t)\left(1 - Q_1(t)\right) x(t), \quad x(0) = x_0, \tag{15}$$

where w_i reflects the firms' attraction efficiency and $G_i(t)$ is the goodwill, in the Nerlove-Arrow sense, of firm $i = 1, 2$. Note that the above dynamics are highly nonlinear, which leads to a model that can only be solved numerically.

Finally, we mention that a series of papers have linked investment in product and process R&D to product differentiation and market proliferation; see, e.g., Cellini and Lambertini (2002, 2004), Lambertini and Mantovani (2009, 2010), and Li (2017). These contributions would fit in our framework if investments in product innovation was interpreted as investment in quality design. To illustrate, Lambertini and Mantovani (2009) and Li (2017) adopt the following dynamics:

$$\dot{d}(t) = d(t)\left(-q^{\mathcal{I}}(t) + \delta\right), \quad d(0) = d_0, \tag{16}$$

where $d \in (0, 1)$ is the degree of product differentiation, and δ is an exogenous technological decay rate. When $s = 1$, the product is homogeneous, i.e., there is no differentiation. In (16), quality improvement is the driver of the dynamics of s. In Cellini and Lambertini (2002, 2004), it is the investment in R&D (or product innovation or design quality) that intervenes in the dynamics, along with the effort conducted by the industry and denoted by $Ind_{R\&D}(t)$. Here, we have

$$\dot{d}(t) = -d(t)\frac{q^{\mathcal{I}}_{R\&D}(t) + Ind_{R\&D}(t)}{1 + q^{\mathcal{I}}_{R\&D}(t) + Ind_{R\&D}(t)}, \quad d(0) = d_0. \tag{17}$$

5 Sales as function of quality

In this section, we report on the impact of quality on sales. As all papers are described in detail in the Appendix, we only give the generic relationships that have been used in the literature and illustrate each one with few examples. We use the following notation:

$S(t)$: cumulative sales at t;

$s(t)$: sales at time t;

$q(t)$: product's quality (control variable);

$Q(t)$: product's quality (state variable);

$p(t)$: product's price;

$A(t)$: advertising effort;

$G(t)$: product's (firm's, or brand's) goodwill;

Further, Greek letters are positive parameters.

First, a series of papers assumes that current sales depend on current quality improvement. For instance, El Ouardighi and Kogan (2013) and Liu et al. (2015a) propose the following form:

$$s(q(t), p(t)) = q(t)\left(\alpha - \beta p(t)\right), \tag{18}$$

which means that quality shifts up the total demand. Liu et al. (2016) retain an additive specification, that is,

$$s(q(t), p(t), q_R(t)) = \alpha - \beta p(t) + \kappa q(t) + \lambda\left(q(t) - q_R(t)\right), \tag{19}$$

where $q_R(t)$ is the reference quality (a state variable). Here, quality increases the market potential α by the amount $(\kappa + \lambda) q(t)$. Kopalle and Winer (1996) introduced the idea of reference quality (see (10)), and it has been used, in one way or another, in Gavious and Lowengart (2012), Liu et al., (2016), Xue et al. (2017), Li (2021), Li et al. (2020), and Ni and Zhao (2021). Finally, De Giovanni and Tramontana (2016) assume that sales depend positively on advertising and conformance quality as follows:

$$s(q(t), A(t)) = \rho A(t) + \kappa q^C(t),$$

where $q^C(t)$ is the rate of defect-free products.

Second, some contributions consider that what influences demand is the stock of quality, that is, a state variable that summarizes all past investments in quality. One example is Chenavaz (2012), where the sales are given by a function $s(Q(t), p(t))$, which is increasing in $Q(t)$ and decreasing in $p(t)$. The analysis is first conducted without assuming any specific forms, and next the results are illustrated with specific relationships, e.g.,

$$s(Q(t), p(t)) = \alpha - \beta p(t) + \gamma Q(t) + \eta \frac{Q(t)}{p(t)}.$$

The last term captures the interaction between quality and prices, which incidentally complicates the analysis. (Li and Ni (2016) set η equal to zero.) Another example is Martín-Herrán et al. (2012) where

$$s(q(t), p(t)) = \alpha - \beta p(t) + \gamma Q(t) - \epsilon (p(t) - p_R(t)),$$

with $p_R(t)$ being the reference price at time t whose evolution is governed by a differential equation. Here, as expected, quality has a positive impact on sales, while the differential term $p(t) - p_R(t)$ positively affects quality and negatively affects demand. That is, if the price is larger than the reference price, then the consumer perceives a higher quality, but this, at the same time, hurts demand. In these contributions, quality is understood in a global sense or as perceived quality. Hirschmann (2014) selects quality experience as an independent variable in the sales function, whereas Mukhopadhyay and Kouvelis (1997), El Ouardighi and Kim (2010), El Ouardighi (2014), Liu et al. (2015b), and El Ouardighi and Shniderman (2019) let the demand depend on design quality stock. To illustrate, El Ouardighi (2014) adopts the following form

$$s\left(Q^{\mathcal{D}}(t), p(t)\right) = \mu Q^{\mathcal{D}}(t) (\alpha - \beta p(t)). \tag{20}$$

De Giovanni and Tramontana (2016) propose a sales function that depends on both advertising and conformance quality strategies. Therefore, when products are known by consumers to conform, sales increase as consumers trust the product quality.

A third approach is to let quality indirectly affect demand, that is, through another driver, typically goodwill. Here, the retained goodwill dynamics extend the model in Nerlove and Arrow (1962) by assuming that it does not only depend on advertising, but also on some measures of quality, that is,

$$\dot{G}(t) = f(A(t), x(t)),$$

where $x(t)$ is either a control variable $q(t)$ or a state variable $Q(t)$, with sales at time t being a function of $G(t)$ and some other variables (typically price). Examples of contributions belonging to this class of model include the following: Buratto et al. (2019) analyze the impact of having a consignment contract in a supply chain; De Giovanni (2011, 2020) assumes that conformance quality $Q^{\mathcal{C}}(t)$ affects the goodwill dynamics, which in turn influences sales; in a competitive setting, Nair and Narasimhan (2006) assume that a firm's goodwill as well as its rival's influence sales, with the goodwill being dependent on quality improvements.

Finally, we mention a series of contributions where the authors included in the model both the rate of sales and cumulative sales at time t; see, e.g., Narasimhan and Ghosh (1996), Teng and Thompson

(1996), Kouvelis and Mukhopadhyay (1995b), El Ouardighi and Tapiero (1998), Mukhopadhyay and Setaputra (2007), Lin et al. (2008), El Ouardighi et al. (2008, 2013, 2016), and Caulkins et al. (2017). Cumulative sales should be considered the decision process when the unit production cost decreases with cumulative production (learning-by-doing benefit), when there is a word-of-mouth effect, or when the market size is fixed and saturation effects are present. To illustrate, Narasimhan and Ghosh (1996) propose the following sales dynamics:

$$\dot{S}(t) = S(t)\left\{\left(1 - \frac{p(t)}{p_C(t)}\right)^{\alpha} + \left(\frac{q^{\mathcal{I}}(t)}{q_C^{\mathcal{I}}(t)} - 1\right)^{\beta} + \left(\frac{A(t)}{A_C(t)} - 1\right)^{\delta}\right\}, \tag{21}$$

where $q_C^{\mathcal{I}}(t)$, $p_C(t)$, and $A_C(t)$ represent competitors' quality, price, and advertising strategies, respectively. Here, the rate of sales $\dot{S}(t)$ is proportional to cumulative sales $S(t)$. Caulkins et al. (2017) assume that the rate of sales evolves as follows:

$$\dot{S}(t) = (\alpha - \beta p(t))\left(q^{\mathcal{I}}(t) - q_R\right) + \gamma A(t) - \delta S(t), \tag{22}$$

that is, the sales dynamics are a function of the price $p(t)$, the instantaneous experience quality $q^{\mathcal{I}}(t)$, the (constant) reference quality q_R, and the advertising rate $A(t)$.

To wrap up, we note that the literature has taken different routes to model the impact of quality on sales, in terms of measuring quality itself (i.e., specific or general, control or state variable), its direct or indirect impact, its role in the set of independent variables, and the functional form of the (rate or cumulative) sales function.

6 Cost of quality

The literature has distinguished between the investment cost to raise product quality and the production cost that may, or may not, depend on the quality level, modeled either as a control or a state variable. The investment cost function to improve product (and occasionally process) quality has been assumed to be convex increasing. As not much can be added beyond that, there is no need to discuss more specifically the contributions in this respect. Further, we disregard all papers where the marginal production cost is independent of the product quality. Generally speaking, in these papers, quality affects sales, but not (at least not directly) the production cost, which solely depends on the quantity produced (see, e.g., Kopalle and Winer (1996), Nair and Narasimhan (2006), Martín-Herrán et al. (2012), Lambertini and Orsini (2015), Li et al. (2020), and Li and Ni (2016)).

A good place to start the discussion on the cost of quality is the economic conformance model (ECM) represented in Figure 1; see Lundvall and Juran (1974) and Juran (1979). According to this model, the optimal level of conformance quality is the result of a tradeoff between the prevention and appraisal costs, given by a decreasing convex function $c_A(q)$, and the cost of failures, given by a decreasing convex function $c_F(q)$. Although this model is intuitive and has been influential, it has been criticized because of its static view of quality and because it does not account for the revenues side of quality, which may explain why a profit-maximizing firm seeks a higher level of conformance quality than q^*.

Fine (1986) is one of the first contributions to extend the ECM to a dynamic setup by considering two types of learning, namely, quality-based learning in manufacturing activities (model I), and quality-based learning in quality control activities (model II). In model I, the marginal production cost is given by

$$C(q,z) = c_A(q) + c_F(q) + c(z),$$

where $c(z)$ is a decreasing function of z given by

$$z(t) = z(0) + \int_0^t s(\tau)\, q(\tau)\, d\tau,$$

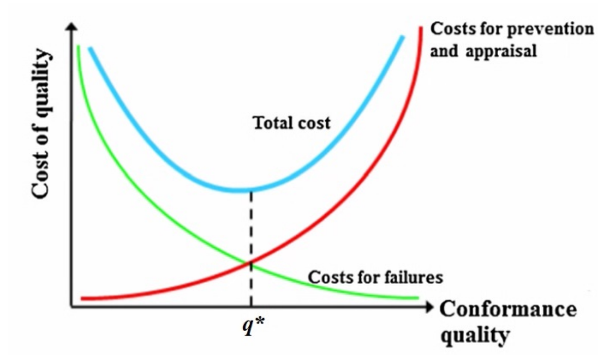

Figure 1: Economic conformance model

with $s(\tau)$ and $q(\tau)$ being the sales and conformance quality at time τ, respectively. In model II, the unit production cost is defined as

$$C(q,z) = f(z)\,c_A(q) + c_F(q) + \alpha, \tag{23}$$

where $f(z)$ is a decreasing function of z, and α is a constant. The optimal quality paths are characterized for the two models, and obviously they are different as they reflect two learning assumptions. Foster and Adam (1996) extended Fine's model by integrating the speed of quality improvement.

Li and Rajagopalan (1998) decomposes the total cost into different pieces representing (1) premanufacturing (which is by definition independent of production), (2) manufacturing, which includes production and quality assurance efforts, and (3) postmanufacturing, which includes customer returns, warranty, and repair efforts. The manufacturing unit cost is specified as follows:

$$C(q + (1-q)\,g(v)), \quad \text{with } \frac{\partial C}{\partial q} < 0 \text{ and } \frac{\partial^2 C}{\partial q^2} \geq 0, \tag{24}$$

where v is the quality assurance effort, and $g(v) \in [0,1]$ is a concave increasing function satisfying $g(0) = 0$ and $\lim_{v\to\infty} g(v) = 1$.

The economic conformance model was integrated into a series of papers, with various specifications of the functional forms of $c_A(q)$ and $c_F(q)$. For instance, Narasimhan and Ghosh (1994) retained

$$C(q) = c_A(q) + c_F(q) = \alpha q^{\beta} + \gamma \frac{1-q}{q}, 0 \leq q \leq 1,$$

whereas Chand et al. (1996) illustrated their fully general results with the following example:

$$C(q) = c_A(q) + c_F(q) = \frac{1}{1+q}.$$

El Ouardighi and Pasin (2006), El Ouardighi et al. (2013), and De Giovanni (2019) adopted a linear conformance cost.

So far, the quality cost has been defined in terms of conformance quality, and the assumption is that the marginal cost decreases with the level of quality. This assumption, which is consistent with the dominant paradigm asserting that non-quality is costly, generally finds strong empirical support (e.g. Ittner et al., 2001).

In a long series of contributions, the focus has been on design quality or on generic quality, i.e., an unspecified type of quality, assuming that the higher the quality, the higher the cost. In, e.g., Ringbeck (1985), Teng and Thompson (1996), and Mukhopadhyay and Kouvelis (1997), the total production cost is given by

$$C(q,s) = c(q)\,s, \tag{25}$$

where $c(q)$ is a convex increasing function. Here, quality is a control variable chosen at each period of time, whereas in, e.g., Kouvelis and Mukhopadhyay (1995b), and Mukhopadhyay and Setaputra (2007), the total cost depends on cumulative quality, that is,

$$C(Q, s) = c(Q)\, s, \tag{26}$$

where $c(Q)$ is a convex increasing function. In (25) and (26), the cost depends on current sales. In El Ouardighi and Tapiero (1998), the marginal production cost is given by the function $C(S, q)$, where S is the cumulative sales and $\frac{\partial C}{\partial S} < 0$, that is, the cost decreases with experience in production. Lin (2008) let the cost also depend on current sales, which makes it possible to distinguish between economies of scale (cost decreasing in output) and learning-by-doing (cost decreasing in cumulative sales). In Vörös (2006, 2019), the cost depends on accumulated quality knowledge, cumulative productivity knowledge, and the level of so-called non-strategic quality attributes.

One stream of the literature associates quality investments to innovation investments. To our knowledge, Wang et al. (2019) is the only paper in the dynamic literature that considers quality a driver for making green products. The quality investments aim at design-for-environment, which lead to green and innovative products (e.g., product innovation). Hence, greener goods aim at decreasing the marginal emission costs: the emissions induced by making goods, ϵ, generate a cost for emissions given by $c_\epsilon \epsilon$. By investing in quality to "green" goods, the marginal emission cost takes the form $c_\epsilon [\epsilon - g_\epsilon q^o(t)]$. In this instance, investments in quality translate into emission abatement.

Chenevaz (2011) introduces a production cost function that is a function of the process innovation investments, with the latter being a proxy for quality. A firm has a certain R&D budget, $\mathcal{B}$, to be used for both product innovation, $q^o(t)$, and process innovation, $\mathcal{B} - q^o(t)$. Therefore, Chenavaz (2011) allows a firm to set the product innovation efforts and then obtain the process innovation investments. The marginal production cost $c(t)$ is a state variable whose evolution depends on process innovation efforts, e.g., R&D investments, and product innovation investments. The implicit assumption is that better production yields higher quality products. For more details on this stream of literature, see, e.g., Bayus (1995), Cohen et al. (1996), Lambertini and Mantovani (2009), Pan and Li (2016), Lambertini et al. (2017), Li and Ni (2018), and Li et al. (2020).

7 Conclusions

This paper surveyed the contributions on quality dynamics both in single-agent and in competitive situations. In this section, we propose some ideas for future investigations, with some of them being relatively straightforward extensions to what has been done, and others being more conceptual (and some time speculative) ones.

Stochastic models: Looking at the "Model" column in the Appendix, we clearly see that the literature has largely adopted a deterministic model. Given that one can hardly argue that e.g., future demands and costs are known with certainty, the choice of a deterministic setup is, at least partially, motivated by mathematical tractability. Still, this choice has allowed researchers to obtain a series of insights into the management of quality over time. Extending most of the proposed deterministic models to a stochastic environment is conceptually easy, but it will come with the cost of losing the opportunity of having closed-form solutions. Also, depending on how uncertainty is introduced, obtaining a numerical solution may be computationally challenging, especially for multi-agent models with strategic interactions. One good place to start the analysis is to consider a two-stage model where quality decision is made once, while other decisions, e.g., pricing and advertising, are made in both periods. This would allow one to gain intuition on how uncertainties in, e.g., demand or cost, affect the quality decision in current and future periods. Next, one can move to a multistage model where quality decision is made in each period.

Frequency of decisions: A common assumption in the literature is that quality can be changed continuously over time. Although it is easy to accept that some control variables, e.g., pricing and advertising, can be continuous, it is harder to consider that product quality, whatever to which Garvin's dimension(s) it refers, can be varied at the same pace. Reddy et al. (2016) argue that it may not be feasible to modify frequently product quality, because of, e.g., the presence of a large fixed cost, and consequently proposed to adopt an impulse optimal control model to deal with this situation. Here, the firm can invest in quality at some instants of time (impulse instants), while other control variables remain continuous. This approach is attractive from methodological and conceptual points of view, but the characterization of the optimal and equilibrium solutions is demanding. If the timing and number of impulses are known, but not their levels, then the determination of the solution (of course numerically) is relatively easy. The difficulty increases significantly when both the timing of impulses and the investment in quality are endogenous. For recent advances on impulse optimal control models see, e.g., Chahim et al. (2012), Perera et al. (2020) and Sadana et al. (2021a, 2021b) for differential games with one impulse player.

Strategic consumers: In many industries, some brands introduce regularly new versions of their products (think about cellphones, laptops, winter coat, etc.). Knowing this, a consumer may act myopically or strategically. A myopic consumer buys the product at the first period at which her utility is positive, whereas a strategic consumer solves an intertemporal optimization problem and purchases the product at the period that gives the highest (positive) utility. The impact of consumer's strategic behavior on the firm's pricing policy has been highlighted in the literature (see, e.g., Farshbaf-Geranmayeh and Zaccour (2021) and the references therein). It would be clearly of interest to incorporate strategic consumers in dynamic (discrete-time) models of quality management to see the dual impact of their behavior on pricing and quality decisions. To proceed, one would need to make an assumption on the distribution of consumers' willingness to pay (a uniform distribution will do), and another one on the relative importance of the two groups of consumers (myopic and strategic) in the market.

Experience and credence products: The true quality of an experience good can only be assessed after consumption, whereas the true quality of a credence product may not be known even after consumption. For these types of goods, perceived quality plays a role in shaping the demand, and the literature has recognized it, essentially by looking at how the perception of quality evolves as function of price (also reference price) and advertising. In an era where consumers are getting more and more information from social medias, it is relevant to consider the impact of such source on perceived quality. In the process, one can also look at how social medias affect imitation buying behavior. For an analysis of quality signalling and imitation behaviors, see the recent contribution in Zhang et al. (2020) and the references therein.

Closing the loop: The literature in all its components, i.e., single firm, oligopoly, and supply chain, focused on sales, and ignored the backward flow, that is, the return of used products to the manufacturer (or to the retailer). Understanding the role of quality in determining the backward flows is more than necessary. The durability, performance, and reliability of a product, to name only these dimensions of Garvin's scale, result from manufacturing decisions and have an impact on the return of used products for recycling, reconditionning or remanufacturing. Put differently, quality decisions do not only affect the forward flow, but also the backward one, and this has clearly an impact on the profitability of operations and emissions of pollutants. Keywords such as *programmed obsolescence*, *green washing*, and *circular economy* have not yet entered the world of full fledged dynamic quality models literature.

R&D versus OM: A series of papers in our survey linked R&D investments to product quality. Investments in R&D aim at either developing a new product (product R&D in the literature), whereas investments in process R&D aim at reducing production cost. Back to our comment on the frequency of quality decisions, it is not realistic to assume that investment in product R&D leads at each instant of time to a new product, while still assuming the same demand function and cost structure. Consequently, the investment in product R&D must be interpreted as an effort aiming at improving the quality of an existing product. Further, the reduction in the production cost due to process R&D has been considered to have no impact on the product quality. This assumption may not hold true if a better process leads to less defects, or if it uses different combination of inputs, e.g., less plastic and more compressed cardboard. In any event, it is worth considering the case where the investment in production process also affects the product quality. This could lead to conceptually richer and possibly more applicable class of models.

Back to Garvin's dimensions: We already mentioned that none of the published paper has integrated all Garvin's dimensions in the model. Performance, feature, experience, and conformance quality have been specifically considered, whereas the other dimensions (i.e., durability, reliability, serviceability, and aesthetics) have been completely disregarded. We suggest some ideas for future developments regarding this second group of dimensions.

Durability is intimately linked to the product life cycle, and therefore to demand replacement and to opportunities for remanufacturing and recycling. Given this interpretation, it may be worth letting durability be a control variable in a dynamic model of quality management. Also, one can raise the following question: would durability become a marketing tool targeting environmentally conscious consumers? Similarly, if environmental concerns lead to frugality, then product durability may rank high on the list of desirable product attributes.

Reliability is the probability that a good does not fail within some time window, and engineers have studied it using stochastic models. The difficulty of solving such models (in closed form) may explain the silence of the literature. One way for making progress is to start by listing the causes, events, and circumstances that make a good less reliable. A dynamic model can next consider some strategies, e.g., investments in artificial intelligence systems, to mitigate these risks and the possible negative consequences. The solution can be constrained to lie in a desirable state, i.e., a state where the probability of failure is below a certain threshold.

Serviceability is part of Total Quality Management. Being largely dematerialized, services are offered on a continuous basis, complement the core product by leveraging its potential, and can be purchased either with the product or afterwards. Level of service could be a control variable that affects directly the demand or a state variable that enters the demand, e.g., the brand reputation.

Aesthetics is the hardest item of Garvin's scale to model, even in a static framework. Assuming that one wants absolutely to consider aesthetics, then one should consider market segments, with each having some specific preferences.

Beyond Garvin's dimensions: Since Garvin's seminal contribution, new dimensions of quality have emerged, thanks to technological development and changing consumer's preferences. We list four of these dimensions without suggesting any specific course of action, but inviting researchers and practitioners interested in quality to attempt to answer the following two questions: (1) How each of these dimensions affects consumer's assessment of product quality, her buying behavior, and ultimately the demand? (2) How each dimension affects the firm's business model? We believe that future modelling effort should think about integrating the following dimensions of quality:

Traceability refers to the availability of information regarding the origins of raw materials, the type of production and procedures adopted, and the transportation modes used.

Authenticity is the possibility for consumers to check the product quality by verifying its originality and identity and avoiding buying (unintentionally) a counterfeit.

Customization is the firm's capacity to adapt its products to consumers' needs. Here the level of quality, defined in terms of, e.g., performance, features and aesthetics, is set by the consumer, which is a clear departure from the literature.

Sustainability and ethics are poised to become important quality criteria when consumers choose a product. The implicit assumption is that consumers appreciate and value products made by using sustainable and ethical procedures and routines, materials and processes, people treatments, salaries, and safety.

Table 2: Summary of the literature review on dynamic quality models

Authors	**Strategic interaction** O (Oligopoly) SC (Supply chain) S (Single agent)	**Model** D Deterministic) S (Stochastic) F (Finite horizon) I (Infinite horizon)	**Decision variables (strategies in games and control variable in single-agent problems)**	**Dynamics**	**Functional form of quality investments / efforts**	**Sales depend on**
Bao and Ma (2017)	O	D, F	Quality and Sales	Sales	Quadratic	Quantity and quality competition
Bayus et al. (1995)	S	D, F	Price, product improvement (quality or product innovation), process innovation	Sales, product improvements, and process innovation	Linear	Stock of process improvement (quality)
Buratto et al. (2019)	SC	D, I	Quality improvement, price, advertising, cooperative programs, price discount	Goodwill	Quadratic	Goodwill, which depends on quality
Caulkins et al. (2017)	S	D, F	Product quality, advertising, price	Sales	Quadratic	Advertising, price, and quality experience.
Carrillo and Gaimon (2000)	S	D, F	Rate of process change, training, preparation change.	Knowledge and production capacity	General form	Production capacity and knowledge change
Cellini and Lambertini (2002)	O	D, I	R&D investment (design quality), quantity	Product differentiation	Constant	Design quality (via R&D) and industry R&D investments
Cellini and Lambertini (2004)	O	D, I	R&D investment (design quality), quantity	Product differentiation	Constant	Design quality (via R&D) and industry R&D investments
Colombo and Lambertini (2003)	O	D, I	Quality (product innovation), advertising	Sales	Not considered	Advertising
Cesaretto et al. (2021)	S	D, I	Quality improvements (technology), pricing, servitization	Goodwill	Quadratic	Goodwill, which depends on quality
Chand (1996)	S	D, F	Production rate, quality improvement efforts	Conformance quality	Linear	Sales = production (decision variable)
Chenevaz (2011)	S	D, F	Product innovation (quality improvements), price	Sales, quality, production cost	Not considered	Price and quality
Chenevaz (2012)	S	D, F	Product innovation (quality), process innovation, price	Sales, quality, production cost	Linear	Price and quality.
Chenevaz et al. (2020)	S	D, S, I	Design quality, price, advertising	Design quality	Linear	Quality, price, and advertising

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Table 2: (continued)

Authors	Strategic interaction O (Oligopoly) SC (Supply chain) S (Single agent)	Model D Deterministic) S (Stochastic) F (Finite horizon) I (Infinite horizon)	Decision variables (strategies in games and control variable in single-agent problems)	Dynamics	Functional form of quality investments / efforts	Sales depend on
Chenavaz and Jasimuddin (2017)	S	D, F	Quality (product innovation), advertising	Quality	Linear	Advertising and quality
Cohen et al. (1996)	S	D, F	Design quality, process innovation	Design quality and process innovation	Linear	Product performance (function of design quality and process dynamics) and competitor product's performance
Dawid et al. (2015)	S	D, I. Stochastic switching time between two epochs.	R&D, production capacity	Knowledge and capacity	Linear and quadratic for knowledge and capacity	Production capacity with horizontal and vertical (quality) differentiation between product versions
De Giovanni (2011)	SC	D, I	Quality improvements, advertising, pricing, cooperative advertising support	Goodwill	Quadratic	Goodwill (function of quality)
De Giovanni (2013)	SC	D, I	Quality improvements, advertising, pricing, cooperative advertising support	Goodwill	Quadratic	Goodwill (function of quality)
De Giovanni and Tramontana (2016)	SC	D, F	Conformance quality, advertising	Conformance quality and advertising	Quadratic	Conformance quality
De Giovanni (2020)	S	D, F	Quality improvement, design quality, advertising, price	Conformance quality and goodwill	Quadratic	Goodwill (function of quality)
De Giovanni (2021a)	SC	D, I	Quality improvements, price, advertising, advertising support	Goodwill	Quadratic	Price and goodwill
De Giovanni (2021b)	SC	D, I	Quality improvements, price	Knowledge	Quadratic	Price
De Giovanni (2021c)	SC	D, I	Design quality, green programs, wholesale price, price	Design quality	Quadratic	Price and design quality

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Table 2: (continued)

Authors	Strategic interaction O (Oligopoly) SC (Supply chain) S (Single agent)	Model D Deterministic) S (Stochastic) F (Finite horizon) I (Infinite horizon)	Decision variables (strategies in games and control variable in single-agent problems)	Dynamics	Functional form of quality investments / efforts	Sales depend on
De Giovanni (2021d)	SC	D, I	Quality improvements, price	Goodwill	Quadratic	Sales depend on goodwill and price
El Ouardighi and Tapiero (1998)	S	D, F	Quality improvement, production	Sales	Not considered	Quality improvements
El Ouardighi and Pasin (2006)	O	D, F	Quality, design quality, advertising, price	Market share, conformance quality, and goodwill	Quadratic	Market share depends conformance quality and goodwill.
El Ouardighi (2014)	SC	D, F	Design quality, price.	Design quality	Quadratic	Price and design quality
El Ouardighi et al. (2008)	SC	D, F	Quality improvement, purchasing rate, advertising, price	Sales, conformance quality, and inventory	Quadratic	Conformance quality (sales decrease with defective items)
El Ouardighi et al. (2013)	SC + O	D, F	Quality improvement, purchasing rate, advertising, price	Sales, conformance quality, and inventory	Quadratic	Conformance quality (sales decrease with defective items)
El Ouardighi et al. (2016)	S	D, I	Quality improvement, advertising.	Sales and conformance quality	Linear and quadratic for quality improvement	Conformance quality and advertising
El Ouardighi et al. (2018)	S	D, F	Quality improvement, advertising, price	Sales and conformance quality	Quadratic	Word-of-mouth
El Ouardighi and Kim (2010)	SC +O	D, F	Design quality, price.	Design quality	Quadratic	Price and competition in design quality
El Ouardighi and Kogan (2013)	SC	D, F	Design quality, quality improvement, price	Conformance quality and design quality	Quadratic for both conformance quality and quality	Price and design quality
El Ouardighi and Shniderman (2019)	SC	D, F	R&D, design quality	Design quality improvement and R&D efforts	Quadratic for design quality improvement	Price and design quality
Fine (1986)	S	D, F	Conformance quality, sales	Learning experience	Not considered	Sales = production

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Table 2: (continued)

Authors	**Strategic interaction** O (Oligopoly) SC (Supply chain) S (Single agent)	**Model** D Deterministic) S (Stochastic) F (Finite horizon) I (Infinite horizon)	**Decision variables (strategies in games and control variable in single-agent problems)**	**Dynamics**	**Functional form of quality investments / efforts**	**Sales depend on**
Foster and Adam (1996)	S	D, F	Conformance quality, sales	Learning experience	Not considered	Sales = production
Fruchter (2009)	S	D, I	Price, advertising	Perceived quality	Quadratic for advertising	Price and perceived quality
Gaimon (1988a)	S	D, F	Rate of acquisition of new capacity, rate of salvage value, price, production rate.	Inventory, production capacity and process innovation	All efforts are linear. Quadratic investment cost in new capacity	Price
Gaimon (1988b)	S	D, F	Rate of acquisition of new capacity, rate of salvage value, price.	Production capacity, level of attributes, and process innovation	Linear	Price and level of attributes (quality of outputs)
Gavious and Lowengart (2012)	S	D, I	Quality, price	Reference quality	Quadratic	Price, quality, and reference quality
Hirschmann (2014)	S	S, I	Quality	Design quality	Linear	Quality
Jackson and Narasimhan (2010)	O	D, F	Quality, price	Quality attractiveness and consumers leaving the market	Not considered	Quality attractiveness, dynamic price-dependent market potential, and the ceasing market
Kogan and Raz (2002)	S	D, F	Quality inspection	Defective items	Not considered	Not considered
Kogan and El Ouardighi (2019)	S	D, F	Quality (induced learning), price	Hazard rate determining the conformance quality rate	Quadratic	Price competition
Kopalle and Winer (1996)	S	D, F	Quality, price	Reference quality and reference price	Quadratic	Quality, price, reference price, and reference quality
Kotowitz and Mathewson (1979)	S	D, I	Quality experience, volumes, advertising	Reference quality	Linear	Sales = production

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Table 2: (continued)

Authors	**Strategic interaction** O (Oligopoly) SC (Supply chain) S (Single agent)	**Model** D Deterministic) S (Stochastic) F (Finite horizon) I (Infinite horizon)	**Decision variables (strategies in games and control variable in single-agent problems)**	**Dynamics**	**Functional form of quality investments / efforts**	**Sales depend on**
Kouvelis and Mukhopadhyay (1995)	S	D, F	Design quality, price	Design quality	Quadratic	Price and design quality
Lambertini (2018)	SC	D, F	R&D effort	Design quality	Quadratic cost function for R&D efforts	Price and quality
Lambertini and Mantovani (2009)	O	D, F	Product innovation, quality improvement, process innovation, R&D	Product innovation, process innovation, and market proliferation	Quadratic for product differentiation, process innovation, and R&D	Production level, product differentiation, and cost efficiency
Lambertini and Mantovani (2010)	O	D, I	Product innovation, process innovation, price/volume	Product innovation and process innovation	Quadratic for product and process innovation	The type of competition (Cournot or Bertrand)
Lambertini and Orsini (2015)	S	D, I	Product innovation, quality improvements, R&D	Quality improvements and production cost	Quadratic for product innovation and R&D	Quality and price
Lambertini et al. (2017)	S	D, I	R&D, process innovation	Quality improvements and production cost	Quadratic for product innovation and process innovation	Quality and price
Li and Rajagopalan (1998)	S	D, F	Process improvement, production, and quality assurance	Knowledge quality and knowledge productivity	General convex form	Sales = production
Li and Ni (2016)	S	D, I	Product innovation, process innovation, quantity	Quality improvements, production cost, and learning-by-doing	Quadratic for product innovation and R&D	Quality and price
Li and Ni (2018)	S	D, F, and I	Product innovation, production	Quality improvements	Quadratic for product innovation	Sales = production
Li et al. (2020)	S	D, I	Product innovation, price, reference quality	Quality improvements, production cost, and reference quality	Quadratic for product innovation, process innovation, and quality improvement	Quality, price, and reference quality

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Table 2: (continued)

Authors	**Strategic interaction** O (Oligopoly) SC (Supply chain) S (Single agent)	**Model** D Deterministic) S (Stochastic) F (Finite horizon) I (Infinite horizon)	**Decision variables (strategies in games and control variable in single-agent problems)**	**Dynamics**	**Functional form of quality investments / efforts**	**Sales depend on**
Li (2020)	S	D, I	Product innovation, R&D	Quality improvements, reference quality and production cost	Quadratic for product innovation and R&D	Quality, price, and reference quality
Li (2017)	S	D, F	Product innovation, R&D	Product and process innovation	Quadratic for product innovation and R&D	Production level, product substitutability, and cost efficiency
Lin (2008)	S	D, F	Quality, price, production rate	Sales and inventory	Not considered	Quality, price, and cumulative sales
Liu et al. (2015a)	S	D, F	Quality improvement, price	Design quality and inventory	Linear	Price and design quality
Liu et al. (2015b)	O	D, I	Quality improvement, advertising, price	Design quality and goodwill	Quadratic	Price, goodwill, and design quality
Liu et al. (2016)	O	D, I	Product quality, price, wholesale price	Product quality (features)	Quadratic	Price, quality, and reference quality
Lu and Navas (2021)	SC	D, F. Time of crisis stochastic	Advertising, quality improvements	Quality and Goodwill	Quadratic	Goodwill, advertising, and quality
Martín-Herrán et al. (2012)	SC	D, I	Quality improvement, price, wholesale price	Brand quality and reference price	Quadratic	Price, reference price, and brand quality
Mendez and Narasimhan (2006)	S	D, F	Quality, price	Quality attractiveness and consumers leaving the market	Not considered	Quality attractiveness, dynamic price-dependent market potential, and the ceasing market
Mukherjee and Carvalho (2021)	SC	D, I	Green quality investments, wholesale price, retail price	Learning, Green quality level	Quadratic	Price, greening quality level
Mukherjee and Chauhan (2021)	O	S, I	Advertising	Goodwill	Quadratic	Goodwill
Mukhopadhyay and Kouvelis (1997)	O	D, F	Quality improvement, price	Design quality and sales	Quadratic	Design quality and price.

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Table 2: (continued)

Authors	Strategic interaction O (Oligopoly) SC (Supply chain) S (Single agent)	Model D Deterministic) S (Stochastic) F (Finite horizon) I (Infinite horizon)	Decision variables (strategies in games and control variable in single-agent problems)	Dynamics	Functional form of quality investments / efforts	Sales depend on
Mukhopadhyay and Setaputra (2007)	S	D, F	Quality, price, refund value	Design quality and sales	Quadratic	Quality, price, and refund policy
Nair and Narasimhan (2006)	O	D, I	Quality improvement, advertising, price	Goodwill	Quadratic	Firm's goodwill and rival's goodwill
Narasimhan and Ghosh (1994)	S	D, I	Quality improvement, advertising, price	Sales	Not considered	Quality improvement, pricing, and advertising
Narasimhan et al. (1993)	S	D, F	Quality, price	Quality attractiveness and consumers leaving the market	Not considered	Quality attractiveness, dynamic price-dependent market potential, and the ceasing market
Narasimhan et al. (1996)	S	D, F	Quality, price	Quality attractiveness and consumers leaving the market	Not considered	Quality attractiveness, dynamic price-dependent market potential, and the ceasing market
Narasimhan and Mendez (2001)	S	D, F	Quality, price	Quality attractiveness and consumers leaving the market	Not considered	Quality attractiveness, dynamic price-dependent market potential, and the ceasing market
Ni and Li (2019)	S	D, F	Price, R&D, product innovation	Quality and goodwill	Linear	Price, goodwill, and quality
Ni and Zhao (2021)	SC	D, I	Product innovation, price	Product quality and reference price	Quadratic	Price, quality, and reference price
Pan and Li (2016)	S	D, F	Product and process innovation, price	Quality and production cost	Quadratic	Quality and price
Reddy et al. (2016)	S	D, F	Quality, advertising	Goodwill, sales	Quadratic	Quality and advertising
Ringbeck (1985)	S	D, F	Price, advertising, quality	Market share	Not considered	Price and market share (function of quality)

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Table 2: (continued)

Authors	Strategic interaction O (Oligopoly) SC (Supply chain) S (Single agent)	Model D Deterministic) S (Stochastic) F (Finite horizon) I (Infinite horizon)	Decision variables (strategies in games and control variable in single-agent problems)	Dynamics	Functional form of quality investments / efforts	Sales depend on
Tapiero (1987)	S	S, F	Quality control procedure, sample size	Learning experience	General form	Sales = production
Teng and Thompson (1996)	S	D, F	Quality improvement, price	Sales	Not considered	Price, quality, and cumulative sales
Vörös (2006)	S	D, F	Quality improvement, non-strategic quality attributes, process improvement, price	Quality knowledge and productivity knowledge	General cost function for design quality and process improvement	Quality knowledge, price, and non-strategic quality attributes.
Vörös (2019)	S	D, F	Quality primary and secondary attributes, price	Quality (design quality)	General investment function for primary attributes	Quality and price
Wang and Li (2012)	S	D, F	Price	Quality (design quality)	Not considered	Quality and price
Wang et al. (2019)	O	D, I	Product innovation, process innovation, price, wholesale price	Product innovation, process innovation, and learning by doing in both	Quadratic for both process and product innovation	Price and product innovation (green quality)
Wang and Hu (2020)	SC	D, I	Quality improvement, promotion	Goodwill	Quadratic	Goodwill and promotion
Xue et al. (2017)	S	D, F	Quality improvement, price	Reference quality and quality	Quadratic for quality improvement	Quality, reference quality, and price
Zhao and Ni (2021)	S	D, I	Quality, sales, emissions abatement	Quality (design quality), cumulative sales, cumulative emissions	Quadratic	Quality, price, and reference price
Zhong and Zhang (2018)	S	D, I	Product innovation, process innovation	Quality improvements, production cost, and learning-by-doing	Quadratic for product innovation and process innovation	Quality and price.

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