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Integrating geometallurgical Ball mill throughput predictions into short-term stochastic production scheduling in mining complexes

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Abstract : The geometallurgical models that predicting the throughput/comminution performance of the a processing plant often rely on rock hardness models, which are based on very sparse information, and are not typically are not designed to interact with mine production scheduling, and while ignoringignore the non-additive nature of hardness. This article presents a novel approach to integrate a geometallurgical throughput prediction model for the ball mill into short-term stochastic production scheduling in industrial mining complexes. The utilized datasets for this prediction model include penetration rates from blast-hole drilling, truck cycle data and measured throughput rates of the operating ball mill, offering an easily accessible and cost-effective method compared to other geometallurgical programs. Firstly, the comminution behavior of the mineral reserve is geostatistically simulated by building additive hardness proportions using blast-hole penetration rates. Then, a material tracking approach considers all material movements in the mining complex to inform the throughput prediction model about rock properties of blended materials sent to the ball mill. Subsequently, a multiple regression model is constructed, which predicts throughput rates as a function of blended rock properties. Finally, the prediction model is integrated into a stochastic integer programming model for short-term production scheduling in mining complexes. A case study at the Tropicana Gold Mining Complex in Western Australia shows that ball mill throughput can be predicted with an error of less than 30 t/h (RMSE) and a correlation coefficient between predicted and observed values of up to 0.8. By integrating the prediction model and newly proposed stochastic components into the optimization, the resulting production schedule can achieve weekly planned production reliably because scheduled materials can now be matched with the predicted performance of the ball mill. Comparisons to optimization using conventional mill tonnage constraints reveal that expected production shortfalls of up to 7 % per period can be mitigated this way.

Keywords: Geometallurgy, stochastic short-term production scheduling, measurement while drilling, non-additivity, hardness

1 Introduction

Short-term mine production planning aims to make daily, weekly, or monthly operational decisions that best meet strategic production targets under existing operating conditions and constraints (Hustrulid et al., 2013). Blom et al. (2018) review past advancements in short-term planning for open pit mines, while recent developments consider the simultaneous stochastic optimization of a short-term mine production schedule and fleet management decisions in mining complexes under supply and equipment performance uncertainty (Quigley and Dimitrakopoulos, 2019; Both and Dimitrakopoulos, 2020). However, the incorporation of pertinent geometallurgical properties into short-term mine production scheduling has not been addressed sufficiently.

Geometallurgy describes the relationships between rock characteristics extracted from mineral deposits and its metallurgical/processing behavior further downstream in a mineral value chain. Key geometallurgical properties should thus be integrated into short-term mine production scheduling, which would lead to better-informed mine plans (Dunham et al., 2011; Dowd et al., 2016). Important properties to consider are the rock hardness and grindability, which largely control the throughput in the comminution circuit of the processing plant(s). The integration of a throughput prediction model is crucial for achieving short-term production targets since plant throughput directly influences metal production. Having a detailed modelling of the comminution circuit in production scheduling is also beneficial from a cost and environmental perspective since a substantial portion of a mine's energy consumption can be attributed to comminution of the ore in hard rock open pit mines (Norgate and Jahanshahi, 2011). Although many geometallurgical models that predict throughput/comminution performance of the mineral reserve based on ore hardness and grindability have been developed (Flores, 2005; Alruiz et al., 2009; Bueno et al., 2015; Ortiz et al. 2020; others), they are typically not designed to interact with mine production scheduling. These geometallurgical throughput models typically rely on very sparse measurements of hardness and typically ignore the non-additive nature of hardness when changing the support-scale from points (measurement scale) to blocks (mining scale), as well as when materials are blended from various sources within a mining complex.

Comminution performance of the mineral reserve is conventionally assessed by different hardness and grindability indices. Lynch et al. (2015) review the most used indices to date, including Bond's Ball Mill Work Index (Wi), the SAG Power Index (SPI), the Drop Weight Index (DWi) and the resistance to impact breakage (A$\times$b). To date, the limited integration of throughput/comminution performance into mine production scheduling involves a spatial model of the mentioned hardness and grindability indices (Coward and Dowd, 2015; Deutsch et al., 2016; Dowd et al., 2016; Morales et al., 2019). The spatial model of geometallurgical properties is built using geostatistical techniques and is commonly referred to as the geometallurgical model. This endeavor is challenging due to the non-additive nature of most hardness and grindability indices (Yan and Eaton, 1994; Amelunxen, 2003; Amelunxen et al., 2014). Specific care has to be taken for spatial interpolation (Deutsch, 2013; van den Boogaart et al., 2013), change of support, including drill-hole compositing and upscaling to mineable units (Garrido et al., 2019; Ortiz et al., 2020), and blending of materials in downstream processes (Newton and Graham, 2011). The latter directly contradicts the common assumptions of mine production scheduling models that use mixed integer linear programming. These models need to assume the additive behavior of variables to enforce blending constraints and processing capacities. The other major challenge is that information about hardness properties is typically derived from very sparse geometallurgical sampling (Hunt et al., 2013; Deutsch et al., 2016). Part of the reason for sparse sampling is that drilling campaigns and laboratory tests on the samples' comminution behavior are typically costly and time-consuming (Mwanga et al., 2015).

Several proxy models have been tested for predicting hardness variables as a function of primary rock attributes, which is also termed a primary-response framework (Coward et al., 2009). Applications include multiple linear regression (MLR) (Keeney and Walters, 2011; Boisvert et al., 2013), projection pursuit regression (Sepúlveda et al., 2017), and a variety of supervised learning techniques (Lishchuk et al., 2019). However, these proxy models are typically constructed using samples in drill-core support and thus do not guarantee the correct mapping of larger volumes (SMU) (Richmond and Shaw, 2009). Furthermore, the calibration of these proxy models is typically performed on an extremely limited number of hardness samples.

In contrast to the above, the use of penetration rates stemming from measurement while drilling (MWD) data is investigated in this article to inform the mineral reserve about hardness and comminution performance. The potential of these spatially dense datasets has already been exploited in mining for enhanced blast design (Segui and Higgins, 2002), the detection of rock fractures (Schunnesson, 1996), rock characterization in various commodities (Schunnesson, 1998; Zhou et al., 2012; Vezhapparambu et al., 2018), and other applications. Although suggested in the technical literature (Segui and Higgins, 2002; Mwanga et al., 2015), MWD has not yet been utilized to create a direct link between the mineral reserve and its comminution performance in milling and grinding circuits. In the approach presented herein, the rate of penetration (ROP) serves as one of the main spatial indicators for ball mill throughput. The utilization of ROP is motivated by the ability to indicate rock-type, strength, and alteration (Horner and Sherrell, 1977; Sugawara et al., 2003; Yue et al., 2004; Vezhapparambu et al., 2018).

The next step in a conventional workflow of integrating throughput into mine production scheduling involves empirically derived process models which are used to calculate the expected mill throughput rates using as input the geometallurgical block model of hardness and grindability indices. Examples of frequently used process models to date are the Bond equation (Bond, 19621), the SMC model (Morrell, 2004), and the CEET model (Dobby et al., 2001). The mill throughput rates are either calculated per resource block or per geometallurgical domain (Alruiz et al., 2009; Coward and Dowd, 2015). Morales et al. (2019) present a stochastic formulation version for long-term single mine plan accounting for uncertain throughput rates through geostatistically simulations. A linear constraint ensures that processed blocks do not exceed the available mill hours per production year. The above efforts towards integrating throughput into mine production scheduling are limited in that the non-additive upscaling behavior of hardness is ignored, and the resulting single-mine production schedule is informed by assessing the mill throughput and economic value per mining block independently. In this way, the schedule ignores that materials are blended and processed together with other materials, which is typical in multi-pit, multi-processor mining complexes. As a result, the non-additive comminution behavior of blended materials in the short-term cannot be captured, and the components of the mining complex are not optimized simultaneously.

As an advantage over the conventional mixed integer linear programming models for production scheduling, the simultaneous optimization of mining complexes successfully takes non-linear blending processes in stockpiles and processing facilities into account to maximize NPV (Montiel and Dimitrakopoulos, 2018; Saliba and Dimitrakopoulos, 2019). Initially developed for long-term planning (Montiel and Dimitrakopoulos, 2015; Goodfellow and Dimitrakopoulos, 2016), simultaneous optimization of mining complexes capitalizes on synergies between decisions that are conventionally optimized separately. Kumar and Dimitrakopoulos (2019) introduce geometallurgical constraints related to hardness in the simultaneous optimization of mining complexes for long-term planning. The constraints aim to achieve a consistent throughput by sending pre-defined ratios of hard and soft rock to the processing plant(s). The mineral reserve is categorized into hard and soft rock by stochastically simulated Wi and SPI values, where the hardness ratios are chosen arbitrarily and do not guarantee optimal material selection for throughput maximization.

In the present article, an alternative approach is proposed which integrates geometallurgical hardness properties into short-term mine production scheduling in mining complexes. More specifically, a ball mill throughput prediction model informed by blended rock attributes related to hardness is constructed, which is then integrated into the simultaneous stochastic optimization of mining complexes. The utilized datasets include penetration rates from blast-hole drilling, truck cycle data, and measured throughput rates of the operating ball mill. Given that all sources of information stem from production data, the novel approach gives a potential financial advantage compared to laboratory grinding tests of ore hardness. Additionally, the geometallurgical response of the processed rock, i.e., throughput, is measured directly in the appropriate support-scale, and non-additivity issues of hardness are alleviated using a compositional approach. Section 2 presents the newly proposed approach including the stochastic integer programming formulation in detail. In Section 3, the proposed approach is applied at the Tropicana Gold Mining Complex in Western Australia. Conclusions follow.

2 Proposed approach

In this section, the four steps required to integrate a geometallurgical throughput prediction model of the ball mill into short-term stochastic production scheduling in mining complexes such as the example shown in Figure 1 are presented. In the first step, the comminution behavior of the mineral reserve is geostatistically simulated by building additive hardness proportions using penetration rates from blasthole drilling. In the second step, a material tracking approach considers all material movements from the various pits to the processing plant, including short-term, run-of-mine stockpiles, to inform the throughput prediction model about properties of blended materials that are sent to the processor. In the third step, measured throughput rates of the operating ball mill are utilized and linked with the previously obtained blended rock characteristics of the processed ore. The link is created through a multiple regression model, which predicts ball mill throughput as a function of rock attributes of blended material. Finally, the proposed throughput prediction model is integrated into short-term production scheduling.

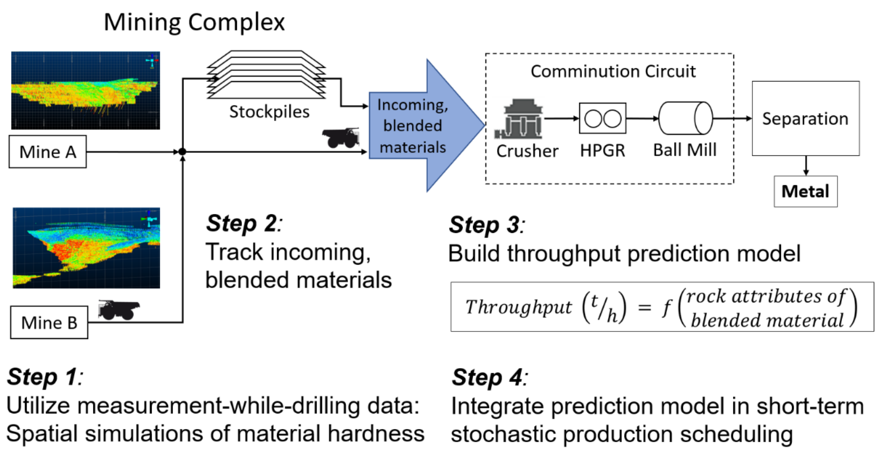


Figure 1: An example of a mining complex and the proposed approach for integrating a geometallurgical throughput prediction model into short-term stochastic production scheduling optimization

2.1 Spatial simulation of material hardness using penetration rates

Measurement while drilling (MWD) data is a centralized collection of mechanical performance indicators obtained from drilling machinery. The monitored performance indicators of the drilling process include, among other entries, rate of penetration, downhole pressure, rotational pressure, rotational speed, air pressure, and vibration. These measurements are routinely collected in operating mines during drilling activities such as exploration, grade control and blast-hole drilling. The latter typically comprises dense drilling patterns (up to 6m $\times$ 6m). A compositional approach of geostatistically simulating the hardness of the mineral reserve is presented next, consisting of creating point simulations of penetration rates first, and creating hardness proportions thereafter.

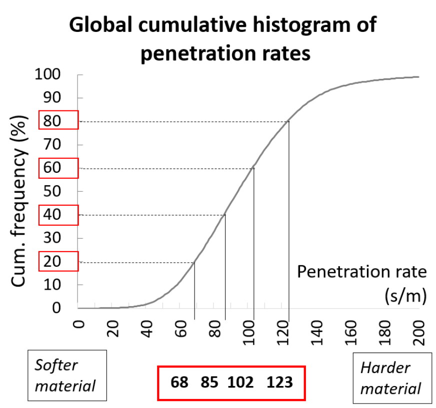
2.1.1 Spatial simulation of drilling rate of penetration

The rate of penetration (ROP) is used in this article for spatial simulation of material hardness. Note that details regarding data cleaning and pre-processing of ROP entries are discussed in the case study presented in Section 3. The ROP measurements are converted to the unit $(\frac{s}{m})$ for down-the-hole compositing. This unit is preferred since compositing (averaging) is performed by down-hole length. The unit $(\frac{s}{m})$ is kept for the remainder of the article. The ROP composites form a set of n conditioning data points distributed in space $\{z_{ROP}(u_\alpha), \alpha = 1, \ldots, n\}$. Conventional geostatistical simulation techniques (Goovaerts, 1997) can be used to create a set of equiprobable spatial simulations of ROP, which is performed on a regularly spaced grid (point support).

A challenge arises for the change of support of hardness-related variables from simulated grid nodes (point support) to mineable volumes (Selective Mining Unit, SMU). Yan and Eaton (1994) indicate that comminution behavior is disproportionally affected by harder fractions of material. Thus, a simple averaging of ROP will create biases (Carrasco et al., 2008; Richmond and Shaw, 2009; Deutsch, 2013; Ortiz et al., 2020). An alternative approach is proposed herein which generates additive geometallurgical, hardness-related variables. Additivity of variables becomes especially important for material tracking and assessing the properties of blended materials, which is discussed in a subsequent section.

2.1.2 Construction of additive hardness proportions

Instead of changing the support scale from grid nodes to mineable volumes, a compositional approach is proposed, which involves the creation of K proportions of softer (easier-to-penetrate) and harder (harder-to-penetrate) material within a larger volume (SMU). Considering all simulated values at the point support scale within a mining block, each proportion represents a fraction of point penetration rates that fall within a specific interval of the simulated ROP. All K proportions naturally sum up to 1, or 100 %, within each mining block. An illustration of hardness proportions is shown in the table on the right in Figure 2. The required intervals are delimited by a set of global ROP thresholds derived from a set of percentiles of the global cumulative histogram of ROP, as seen on the left in Figure 2. The number of required intervals (percentiles) is discussed in the case study.



Block ID	$< 68\ \frac{s}{m}$	[68 ... 85)	[85 ... 102)	[102 ... 123)	$\geq 123\ \frac{s}{m}$
1	0.20	0.22	0.16	0.22	0.20
2	0.64	0.16	0.14	0.06	0.00
3	0.05	0.05	0.22	0.38	0.31
4	0.44	0.19	0.22	0.11	0.05
5	0.41	0.39	0.13	0.08	0.00
6	0.13	0.13	0.19	0.36	0.20
7	0.59	0.20	0.17	0.03	0.00
8	0.42	0.27	0.17	0.13	0.02
9	0.05	0.14	0.27	0.33	0.22
10	0.28	0.14	0.19	0.25	0.14

Figure 2: Construction of penetration rate thresholds (left), used to create hardness proportions per mining block (right)

2.2 Tracking blended materials through the mining complex

A main challenge for linking the simulated rock characteristics of the mineral reserve to the observed production data at a processing plant is to keep track of the materials extracted from the pits and sent to the processing facilities, as indicated in the second step of the proposed framework (Figure 1).

The aim of this step is to characterize the pertinent rock attributes of blended materials entering the comminution circuit so that they can be matched later with observed responses of the ball mill. In addition to the created hardness proportions, other orebody attributes, such as rock density and rock type can be tracked. The material tracking approach is schematically shown in Figure 3. The design accounts for multiple mines and includes all short-term stockpiling that occurs before materials enter the processing facilities.

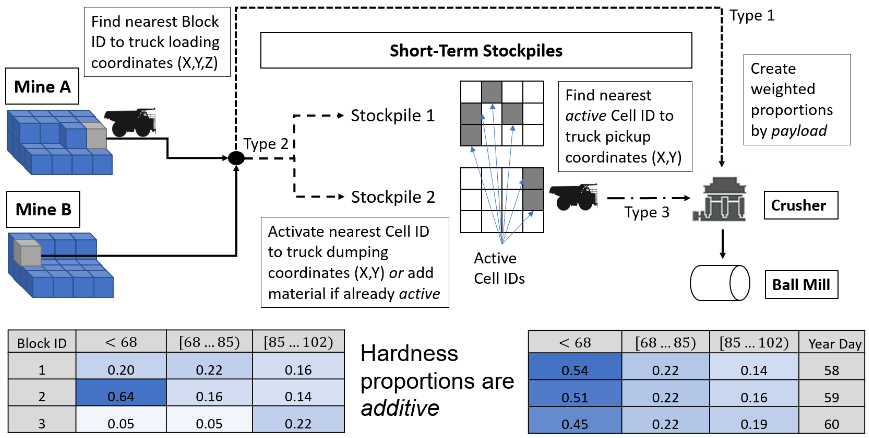


Figure 3: Schematic representation of material tracking of hardness proportions through the mining complex, including short-term stockpiles

The material movements in the various pits and stockpiles are replicated in detail using truck cycle data. The necessary information includes the start and end times of truck cycles, as well as spatial coordinates of truck loading and dumping locations. Three types of truck cycles are relevant for material tracking from the mines to the processing facilities: (1) ore is hauled directly from one of the pits to the crusher; (2) ore is hauled from one of the pits to one of the short-term stockpiles; (3) ore is reclaimed from a short-term stockpile and transported to the crusher. In the presented material tracking scheme (Figure 3), all truck cycles starting in the mine (Type 1 and 2) are first linked to their nearest Block ID of the simulated resource model (hardness proportions, rock type, density) using a nearest-neighbor search. Cycles that end at the crusher location (Type 1 and 3) are utilized for calculating properties of incoming, blended materials.

While Type 1 cycles are easy to trace back to the resource model, a more elaborate tracing scheme is constructed for stockpiled material (Figure 3). Other than blending beds or homogenization facilities (Kumral, 2006), short-term stockpiles (run-of-mine pads or fingers) may not have a fixed build/remove scheme, and their position and dimensions change over time. Thus, an initial grid of cells (10m $\times$ 10m) is constructed, one for every stockpile, covering the area of potential material placement. When a truck dumps material on one stockpile (Type 2), the closest cell is determined by a nearest neighbor search using its dump coordinates. If this cell has been active before (previously dumped material), the associated Block ID of the truck cycle is added to a list. Otherwise, the cell is activated, and the associated Block ID of the truck cycle becomes its first entry. When material is hauled from a stockpile to the crusher (Type 3), the closest active Cell ID is found using truck loading coordinates. The respective list of Block IDs informs about all material that has been deposited there for calculation of blends (one-to-many relationship, detailed in Wambeke et al. 2018). An action of deleting all active cells in an individual stockpile is triggered after a stockpile is depleted.

Weighted proportions of all tracked properties are constructed by truck payload (weighting factor) from all Type 1 and Type 3 cycles that are recorded in a specific interval arriving at the crusher. The time interval for material tracking can be as small as a few minutes or can comprise several days, constrained by minimum truck dumping frequency recorded in the fleet database.

2.3 Building a throughput prediction model

A multiple linear regression model is constructed to predict ball mill throughput (response variable) as a function of rock properties of blended materials (predictor variables or features), which have been obtained previously in the material tracking step. In the regression model, the i^{th} observed throughput, y_i, in a sample of N observations, is expressed as a linear combination of M predictor variables of the tracked rock properties, $f_{i,j}$, as shown in Equation 3.

$$Throughput_i\left(\frac{t}{h}\right) = y_i = w_0 + \sum_{j=1}^{M} w_j \cdot f_{i,j} + e_i \tag{3}$$

Note that w_j, $j = 0, 1, \ldots, M$, are the regression coefficients, and e_i is a random variable representing the prediction error. The observed throughput rates, y_i, are average observed ball mill throughput rates (milled ore tonnage per operating hour), collected for the same time intervals as the tracked rock properties, $f_{i,j}$. The goal of regression is to find the vector of weights, $\mathbf{w} = [w_0, w_1, \ldots, w_M]$, that minimize the sum of squared errors (SSE),

$$SSE(\mathbf{w}) = \sum_{i=1}^{N} e_i^2 = \sum_{i=1}^{N} \left(\mathbf{w}^T \mathbf{f}_i - y_i\right)^2 \tag{4}$$

where $\mathbf{f}_i = [1, f_{i1}, \ldots, f_{iM}]$ is a vector of predictor variables for the i^{th} observation. The closed-form solution of the minimization of SSE with respect to the vector of weights, written in matrix form, is

$$\mathbf{w} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y} \tag{5}$$

(Everett and Howard, 2011; Rencher and Christensen, 2012), where $\mathbf{X} \in \mathbb{R}^N \times \mathbb{R}^{M+1}$ is the feature matrix and $\mathbf{y} \in \mathbb{R}^N$ is the vector of all observations. The resulting weights, $\mathbf{w}^* = [w_0^*, \ldots, w_M^*]$, are used as input for production scheduling. A discussion of feature selection and regression results, including an analysis of correlation to ball mill throughput, are presented in Section 3.

2.4 Integration of throughput prediction model into stochastic short-term mine planning

The mathematical model which integrates the throughput prediction model into the simultaneous stochastic short-term optimization of mining complexes is presented next. The objective function (Equation 6) is formulated as a two-stage stochastic mixed integer program with recourse. All utilized indices, variables, and parameters are listed in Tables 1 to 3, respectively. New parts related to the integration of the developed ball mill throughput prediction model are discussed as follows.

$$\begin{aligned}
\text{Maximize}\quad & \underbrace{\frac{1}{\mathbb{S}} \sum_{j\in\mathbb{J}} \sum_{l\in\mathbb{L}} \sum_{s\in\mathbb{S}} \sum_{t\in\mathbb{T}} p_{j,l} \cdot v_{j,l,s,t}}_{Part\ 1} \\
& - \underbrace{\frac{1}{\mathbb{S}} \sum_{j\in\mathbb{J}} \sum_{l\in\mathbb{L}} \sum_{s\in\mathbb{S}} \sum_{t\in\mathbb{T}} \left({c_{j,l}}^{+} \cdot d^{+}_{j,l,s,t} + {c_{j,l}}^{-} \cdot d^{-}_{j,l,s,t}\right)}_{Part\ 2} \\
& - \underbrace{\frac{1}{\mathbb{S}} \sum_{l\in\mathbb{L}} \sum_{s\in\mathbb{S}} \sum_{t\in\mathbb{T}} \left({c_l}^{TPH+} \cdot d^{TPH+}_{l,s,t} + {c_l}^{TPH-} \cdot d^{TPH-}_{l,s,t}\right)}_{Part\ 3} \\
& - \underbrace{\sum_{t\in\mathbb{T}} \sum_{b\in\mathbb{B}} c_{smooth} \cdot y_{b,t}}_{Part\ 4}
\end{aligned} \tag{6}$$

The main goals of the objective function (Equation 6) are the maximization of profit generated in the mining complex while minimizing the risk of not meeting production targets which is a key objective for short-term plans. Part 1 of the objective function sums all revenues and costs generated within the mining complex. Part 2 is a stochastic component that penalizes deviations from production targets given orebody uncertainty. Both parts are well established building blocks of simultaneous stochastic optimization of mining complexes, both in long-term (Goodfellow and Dimitrakopoulos, 2016) and short-term planning (Villalba Matamoros and Dimitrakopoulos, 2016; Both and Dimitrakopoulos, 2020). Part 3 is a new component introduced in this article and penalizes deviations from the mill tonnage that can be processed based on the throughput that is calculated using the proposed prediction model. Part 4 ensures that the resulting production schedule is mined in a connected (smooth) pattern. Details of the new constraints related to ball mill throughput are given below. Other included constraints are mine capacity constraints, blending constrains, reserve constraints, slope constraints, destination constraints, and material flow constraints, which are detailed in Goodfellow and Dimitrakopoulos (2016, 2017). Note that the reserve constraints in this formulation enforce all blocks to be extracted by the end of the short-term planning horizon.

Table 1: Notations for indices and sets in the mathematical model

$t \in \mathbb{T}$	Index of a time period for the short-term planning horizon $\mathbb{T}$
$s \in \mathbb{S}$	Index of an orebody scenario in the set of orebody scenarios $\mathbb{S}$
$l \in \mathbb{L}$ $\mathbb{L} = \mathcal{A} \cup \mathcal{P} \cup \mathcal{S} \cup \mathcal{W}$	Index of a location l in a mining complex, which is a joint Set of Mining Areas, $\mathcal{A}$, Processing facilities, $\mathcal{P}$, Stockpiles, $\mathcal{S}$, and Waste Dumps, $\mathcal{W}$
$j \in \mathbb{J}$ $\mathbb{J} = \mathbb{H}_{TPH} \cup \mathbb{H}_M \cup \mathbb{H}_T$	Index of an attribute j in the mining complex in the joint set of throughput-related attributes ($\mathbb{H}_{TPH}$) metal-related attributes ($\mathbb{H}_M$) and tonnage-related attributes ($\mathbb{H}_T$)
$b \in \mathbb{B}$	Index of a mining block in the set of all blocks $\mathbb{B}$
$g \in \mathbb{G}$	Index of a group g of material in the set of all material groups $\mathbb{G}$

Table 2: Variables in the mathematical model

$d^+_{j,l,s,t}$, $d^-_{j,l,s,t}$	Continuous variables modelling either surplus (+) or shortage (−) of attribute $j \in \mathbb{H}_M \cup \mathbb{H}_T$ at location l in period t for orebody scenario s
$d^{TPH+}_{l,s,t}$,$d^{TPH-}_{l,s,t}$	Continuous variable modelling positive (+) and negative (−) deviations from the throughput that can be realized in the location l in period t for orebody scenario s
$v_{j,l,s,t}$	Continuous variable which represents the quantity of an attribute j in location l for scenario s in period t
$x_{b,t}$	Binary variable which equals 1, if block b is mined in period t, 0 otherwise
$y_{b,t}$	Integer smoothing variable which reflects the number of adjacent blocks of block b that are not extracted in the same period t
$z_{g,l,t}$	Binary variable which equals 1 if a group of material g is sent to location l in period t, 0 otherwise
$\delta_{l,l',t}$	Continuous variable [0,1] which represents the proportion of material sent out from location l to a receiving destination l' in period t

Complementing Part 3 of the objective function, new components are added to integrate the developed throughput prediction model into the simultaneous optimization of mining complexes. Consider the periods t for a short-term horizon $\mathbb{T}$. The resulting short-term production schedule is informed by a set of orebody simulations, $\mathbb{S}$, containing attributes that are used for throughput evaluation, $j \in \mathbb{H}_{TPH}$, including hardness proportions, and other attributes important for scheduling, such as metal grades. All throughput-related attributes of blended materials sent to the processing location, $l \in \mathcal{P}$, in period t, and orebody scenario s, are represented by $v_{j,l,t,s}$. This variable is calculated as a weighted average of block attributes, $\beta_{b,j,s}$, extracted in t by their respective tonnage, Ton_b, and sent to the processor l, shown in Equation 7.

Table 3: Notations for parameters in the mathematical model

$p_{j,l}$	The unit price for attribute j in location l, which can be positive (revenue) or negative (cost)
${c_{j,l}}^{+},\ {c_{j,l}}^{-}$	Penalty cost of positively (+) or negatively (−) deviating from the attributes $j \in \mathbb{H}_M \cup \mathbb{H}_T$ at location l
${c_l}^{TPH+}, {c_l}^{TPH-}$	Penalty cost of positively (+) or negatively (−) deviating from the throughput that can be realized in the location l
c_{smooth}	Penalty cost for unconnected blocks, enforcing the extraction of blocks in a connected pattern
$\beta_{b,j,s}$	Value of attribute j for block b in orebody scenario s, including metal content, hardness proportions (fractional), degree of weathering (fractional), and density
$\theta_{g,b,s}$	Parameter indicating that block b belongs to material group g in orebody scenario s
Ton_b	Tonnage of block b
$Ton_{l',t-1}$	Tonnage of stockpile $l' \in \mathcal{S}$ in the previous period $(t-1)$

$$v_{j,l,s,t} = \frac{\sum_{g\in\mathbb{G}} \sum_{b\in B} x_{b,t} \cdot \beta_{b,j,s} \cdot Ton_b \cdot \theta_{g,b,s} \cdot z_{g,l,t} + \sum_{l'\in\mathcal{S}} v_{j,l',s,t-1} \cdot Ton_{l',t-1} \cdot \delta_{l',l,t}}{\sum_{b\in B} x_{b,t} \ \cdot \ \mathrm{Ton}_b + \sum_{l'\in\mathcal{S}} Ton_{l',t-1} \cdot \delta_{l',l,t}}$$
$$\forall t \in \mathbb{T}, s \in \mathbb{S}, l \in \mathcal{P}, j \in \mathbb{H}_{TPH} \quad (7)$$

The formulation also considers material attributes sent from locations other than mines, $l' \in \mathcal{S}$, i.e., stockpiles and external sources, to the processor l. The throughput calculation for a processing location for any given period t and orebody scenario s at location l is shown in Equation 8.

$$Throughput_{l,s,t}\left(\frac{t}{h}\right) = w_0^* + \sum_{j\in\mathbb{H}_{TPH}} w_j^* \cdot v_{j,l,s,t} \qquad \forall t \in \mathbb{T}, s \in \mathbb{S}, l \in \mathcal{P} \quad (8)$$

Throughput rates are calculated utilizing the optimized weights, $\mathbf{w}^*$, obtained in Equation 3. Specifically, the prediction model is queried at every iteration of the metaheuristic solution method, using the current production schedule. The throughput prediction obtained in Equation 8 is used to assess the tonnage that can be processed by the ball mill, as shown in Equation 9.

$$Mill\ Tonnage_{l,s,t}^{processed} = Throughput_{l,s,t} * h_l^{avail} \qquad \forall t \in T, s \in S, l \in \mathcal{P} \quad (9)$$

Available hours of the ball mill, h_l^{avail}, are obtained by the use of an empirical availability factor, representing scheduled and unscheduled downtime of the comminution circuit. As the last component, a stochastic constraint is added to the mathematical model as shown in Equation 10.

$$Mill\ Tonnage_{l,s,t}^{processed} - Mill\ Tonnage_{l,s,t}^{scheduled} - \ d_{l,s,t}^{TPH+} + \ d_{,s,t}^{TPH-} = 0 \quad \forall t \in T, s \in S, l \in \mathcal{P} \quad (10)$$

This constraint penalizes a deviation between the scheduled tonnage sent to the processing unit and the calculated tonnage in Equation 9. The latter tonnage represents the ore tonnage that can actually be processed by processing location l, which is now based on pertinent rock attributes related to hardness.

The above approach of modelling mill constraints stands in contrast to conventional tonnage constraints which do not take the hardness properties of blended materials into account. The deviation variables $d^+_{TPH,t,s}$ and $d^-_{TPH,t,s}$ are penalized in Part 3 the objective function (Equation 6), similar to previously developed stochastic constraints for stochastic mine planning (Ramazan and Dimitrakopoulos, 2005, 2013).

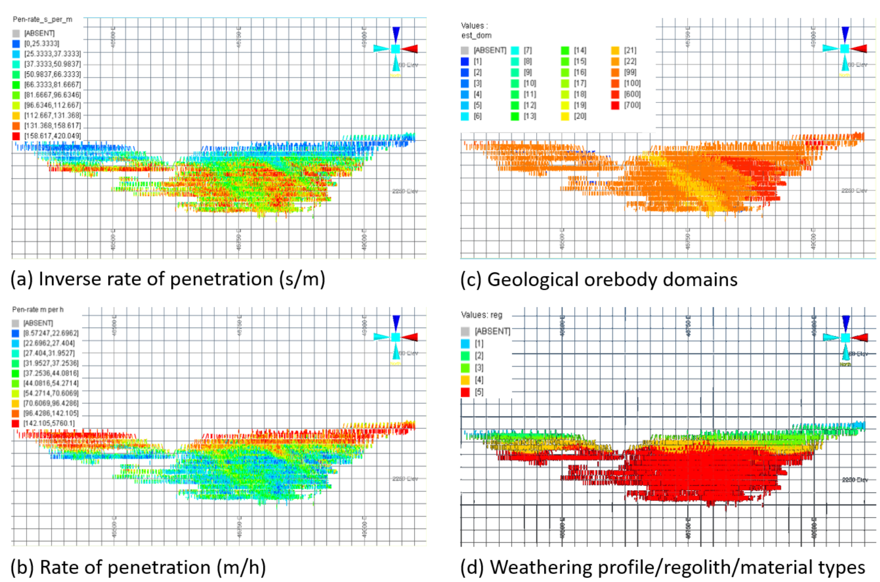
3 A case study at the Tropicana Gold Mining Complex

In this section, the proposed approach of integrating a ball mill throughput prediction model into simultaneous stochastic short-term production scheduling is applied at the open pit Tropicana Gold

Mining Complex, which is located 330km East-North-East of Kalgoorlie in Western Australia. The gold mining complex comprises four contiguous pits extending six kilometers in strike length from North to South. Ore is sent to a single processing facility, consisting of a comminution circuit and a carbon in leach (CIL) facility. The comminution circuit comprises a primary crusher, a high-pressure grinding role (HPGR), and a ball mill. In this case study the material to be extracted on the short-term horizon of one year is mined from two of the four pits, located in distinct mining areas within each pit. The first six months of production data (blast-hole drilling, truck cycle data, and measured ball mill throughput rates) are used to inform the throughput prediction model, whereas three months out of the latter half will be used for production scheduling. The gold mining complex is operated as a typical truck and shovel open pit operation. After fragmentation through conventional drilling and blasting, ore is hauled by truck directly to the crusher or to one of eight short-term, run-of-mine (ROM) stockpiles located in the vicinity of the primary crusher. Temporarily stockpiled material amounts to 80–90 % of processed ore, thus, it is crucial to include material of ROM stockpiles in material tracking, as shown in the case study (Section 3.2).

3.1 Hardness proportions

The raw, recorded penetration rates used to create hardness proportions stem from MWD data collected by all drilling activities (blast-hole, grade control, pre-split drilling) routinely performed in the mining complex. Figure 4 presents cross-sections of the recorded rate of penetration (ROP), shown in seconds per drilled meter (inverse penetration rate, Figure 4a), and drilled meters per hour (Figure 4b), which can be visually compared to the geological domains of the orebody (Figure 4c) and the weathering profile/regolith/material types of the deposit (Figure 4d). Large-scale geological structures, such as dipping geological units displayed in Figure 4c, can be clearly distinguished by consistent differences in ROP. Furthermore, the five stratigraphic regolith units or material types of the orebody (weathering profile, Figure 4d) are strongly reflected in ROP, indicating weathered, easier-to-penetrate rock towards the surface (blue colors in Figure 4a, red colors in Figure 4b). Also, the harder-to-penetrate, fresh rock, situated in greater depth, is easily detectable by penetration rates (red colors in Figure 4a, blue-green colors in Figure 4b).



(a) Inverse rate of penetration (s/m)

(c) Geological orebody domains

(b) Rate of penetration (m/h)

(d) Weathering profile/regolith/material types

Figure 4: Spatial visualization of penetration rate (a, b) retrieved from measurement while drilling data, compared to geological domains (c) and weathering profile (d) of the orebody

It is important to note that biases in this database may exist due to several factors, including the utilization of different drill rig types, multiple rig operators, the degree of bit-wear, and varying drilling tasks. Thus, the data can contain entries that are less correlated to properties of the penetrated rock but rather relate to operational procedures. Many normalization procedures have been proposed in the technical literature (Teale, 1965; Schunnesson, 1998; Zhou et al., 2012). However, these normalization procedures need to be applied carefully. Internally created variables of the MWD system in the present dataset, such as the Blastability Index (BI) (Segui and Higgins, 2002), were found to be highly biased towards individual drilling machines. Furthermore, the application of the adjusted penetration rate (APR) (Zhou et al. 2012) resulted in new, machine-related biases and reduced geological correlation compared to ROP. This shows that site-specific data handling is required. The following targeted outlier removals of ROP data entries were applied on the present dataset, proving to be effective in this case study:

- Production drilling was utilized exclusively (blast-holes), removing pre-split and grade control drilling from the dataset. The reason for this is that blast-hole drilling is conducted solely with drill rigs of the same type in this mining complex.
- Due to pre-fragmented rock from previous blasting activities (easier to penetrate, resulting in higher ROP), the first 2m of every hole were discarded.
- Data entries due to some irregular events (enlarging the drill rod, flushing the hole, etc.) were identified and removed.

3.2 Material tracking in the mining complex

A fleet management system installed in the mining complex records each individual truck cycle in a central database and can therefore be used to replicate material movements in the whole mining complex, as described in Section 2.2. Figure 5 visualizes how material tracking in short-term stockpiles (Run-of-Mine Fingers) is performed at the mining complex.

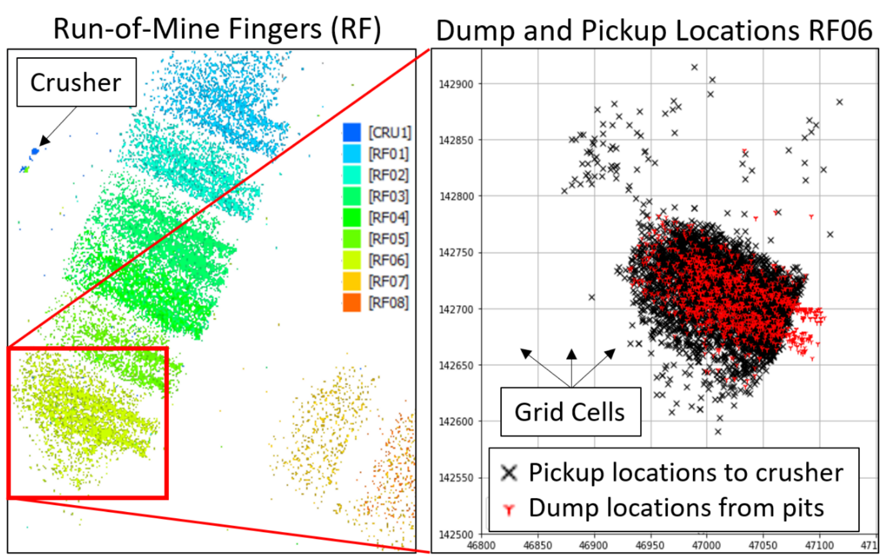


Figure 5: Top view of material tracking in short-term stockpiles (Run-of-Mine Fingers) (left), and close-up of a single stockpile (RF06) (right)

The left-hand side in Figure 5 shows a top view of all ROM Fingers (stockpiles), located in the vicinity of the primary crusher. A close-up of the sixth stockpile (RF 06) shows all dumping events of stockpiled material coming from pits (red markers) and reclaimed material sent from the stockpile to the crusher (black markers). Furthermore, the created grid cells are indicated, which are used to spatially inform and distinguish material within the stockpiles depending on where it has been deposited.

3.3 Multiple regression implementation and parameter testing

The material tracking is conducted in daily time intervals for a period of one year. As part of data preparation for multiple regression, a moving average is calculated for both blended rock properties (predictor variables) and observed ball mill throughput (response variable) so that similar time intervals are preserved for inputs and outputs of the regression. Figure 6 shows the application of a seven-day moving average on daily average ball mill throughput observed at the mining complex.

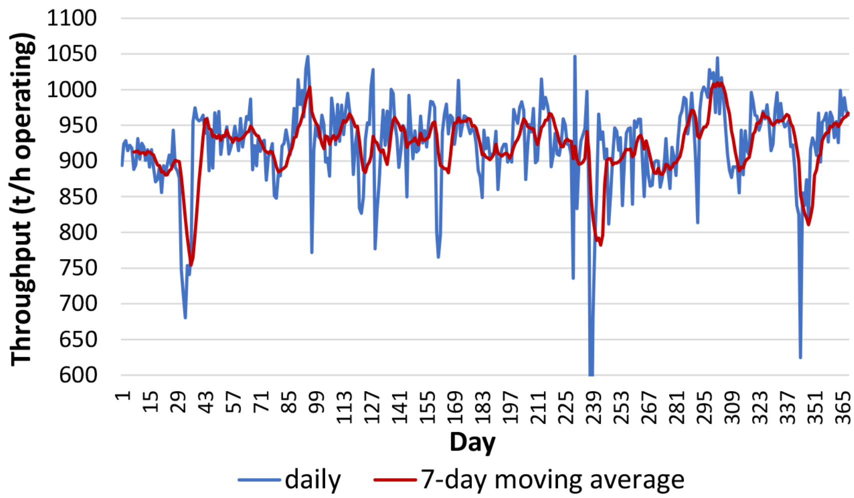


Figure 6: Observed ball mill throughput showing daily average(blue) and 7-day moving average (red)

The moving average is applied because the daily throughput rates observed at the mine show very noisy behavior (blue line). It can be seen in Figure 6 that a seven-day moving average helps emphasize trends of higher and lower throughput rates over longer periods, which are more likely connected to rock properties of the processed material. At the same time, the influence of technical disruptions and short-term delays of ore feed is reduced. Additionally, an important reason for applying a seven-day moving average is that the prediction model is aimed to predict a ball mill throughput for production scheduling comprising weekly periods. By applying the seven-day moving average, data fits the time scale on which ball mill throughput predictions will be used for in production scheduling.

From the available 365 days, a 180-day window is retained for regression analysis, discarding data entries in the beginning and end of the observed interval. This ensures that (i) material deposited in the short-term stockpiles is known, and (ii) material in later time intervals is retained so that it can be used for production scheduling. Correlations between rock attributes of blended materials and observed ball mill throughput are shown in Table 4 using Pearson's correlation coefficient (11),

$$r = \frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)\left(y_i - \overline{y}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2} * \sqrt{\sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2}} \tag{11}$$

with x_i and y_i individual sample points and $\overline{x}$, $\overline{y}$ indicating the respective sample means.

An important observation of Pearson's correlation coefficients shown in Table 4 is that the strongest linear relationships exist between the constructed hardness proportions (A1–A10) and ball mill throughput. The softer hardness proportions (easier-to-penetrate, A8–A10) correlate positively with ball mill throughput, indicating that the throughput increases when more of this softer material is processed in the ball mill. On the other hand, increased proportions of harder-to-penetrate materials lead to a decreased ball mill throughput, indicated by a negative correlation coefficient (A1–A6). The consistency of positive and negative correlation among hardness proportions supports the hypothesis that penetration rates recorded by drilling machines have the capacity to predict comminution performance/grindability of the rock. Other correlation coefficients in Table 4 indicate that the originating geological orebody domains (B1–B4) and average material density (D1) have less influence on ball mill throughput in this case study. However, the different degrees of weathering/regolith/material types (C1–C2) show a stronger correlation to throughput, whereas higher proportions of weathered material are positively correlated with ball mill throughput. This result is not surprising since higher degrees of weathering can cause degradation in rock competency, leading to better grindability in the comminution circuit (Bhuiyan et al., 2019).

Table 4: Pearson's correlation coefficient between rock attributes of blended materials processed in the comminution circuit and observed ball mill throughput

No.	Category	Feature	Unit	Pearson's correlation coefficient: [-1,1]
A1	(harder material)	> 139 s/m $\Leftrightarrow$ < 26 m/h	(%)	−0.274
A2	Material hardness using proportions of penetration rate categories	123…139 s/m $\Leftrightarrow$ 26…29 m/h	(%)	−0.370
A3		112…123 s/m $\Leftrightarrow$ 29…32 m/h	(%)	−0.525
A4		102…112 s/m $\Leftrightarrow$ 32…35 m/h	(%)	−0.361
A5		94…102 s/m $\Leftrightarrow$ 35…38 m/h	(%)	−0.416
A6		85…94 s/m $\Leftrightarrow$ 38…42 m/h	(%)	−0.318
A7		78…85 s/m $\Leftrightarrow$ 42…46 m/h	(%)	0.028
A8		68…78 s/m $\Leftrightarrow$ 46…53 m/h	(%)	0.356
A9		58…68 s/m $\Leftrightarrow$ 53…62 m/h	(%)	0.386
A10	(softer material)	< 58 s/m $\Leftrightarrow$ > 62 m/h	(%)	0.498
B1	Proportions of originating ore domains	Fraction of domain 22	(%)	0.113
B2		Fraction of domain 6	(%)	−0.074
B3		Fraction of domain 600	(%)	0.161
B4		Fraction of domain 99	(%)	0.046
C1	Weathering proportions	Fraction of weathered rock	(%)	0.341
C2		Fraction of fresh rock	(%)	−0.341
D1	Density	Average material density	(t/m^3)	−0.086
Correlation to: Ball Mill Throughput			(t/op.h)	

3.3.1 Number of hardness proportions

The construction of hardness proportions requires a choice of the number of thresholds and their distribution of percentiles, which can be chosen to be evenly or unevenly distributed. Figure 7 shows variations of splitting penetration rates into different hardness proportions and their effect on the multiple regression using only the constructed proportions as predictor variables (features). Leave-one-out cross-validation is applied (Hastie et al., 2009), which consecutively builds a regression model using all data points but one, then predicts the value of the left-out data point. This process is repeated for every data point.

A first result of regression analysis in Figure 7 is that hardness proportions enable a prediction of ball mill throughput (orange line) with a root mean squared error (RMSE) of less than 30 t/h. Additionally, it can be observed that splitting the cumulative histogram of penetration rates differently (even split, soft tail, hard tail) leads to different prediction behavior. The split of softer material appears to predict medium/high observed throughput rates better, whereas the split of harder material clearly predicts lower throughput rates better. Note that average-type approaches of hardness relying on single valued hardness indicators such as Wi and SPI cannot capture these observed differences, which are driven by varying proportions of harder and softer rock. When comparing the five hardness and ten hardness proportions, splitting into more categories appears to have lower RMSE.

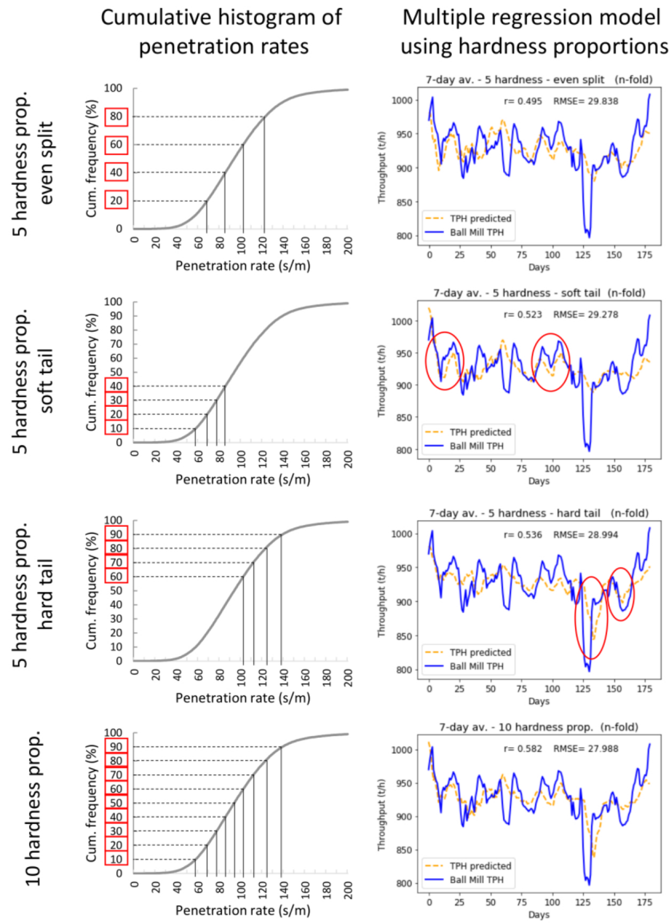


Figure 7: Regression analysis testing multiple splits of hardness proportions

3.3.2 Generalization potential of various features

The material density, geological orebody domains, and degree of weathering, are also tracked next to hardness proportions, as listed in Table 4. A comparison of their potential to predict ball mill throughput in the present case study is shown in Figure 8. Here, the various regression models are built on a data set comprising 80% of data points. The models are then tested on the remaining 20% of data points, revealing the generalization potential to unseen data of the tested feature sets. Note that testing on a consecutive time series of throughput data is more informative than a randomly picked test set since random test samples can be very similar to neighboring training sample points. Also, note that the y-axis changes the scale for some graphs in Figure 8 depending on the spread of throughput predictions.

It can be observed that the use of five hardness proportions shows the best results among all tested cases, shown in Figure 8a (even split), Figure 8b (soft tail), and Figure 8c (softer tail). The splitting the soft tail of the penetration rate distribution shows the highest correlation coefficient of 0.80 (Figure 8b), whereas splitting the harder tail results in the lowest RMSE of 21.88 t/h (Figure 8c).

As expected from correlation analysis, adding the average rock density of blended materials (Figure 8e) as well as adding geological domains as features (Figure 8g) do not influence prediction capability. Interestingly, the addition of the weathering degrees (regolith/material types) as features lead to a higher correlation between test data and predictions (Figure 8f). This can be explained by the

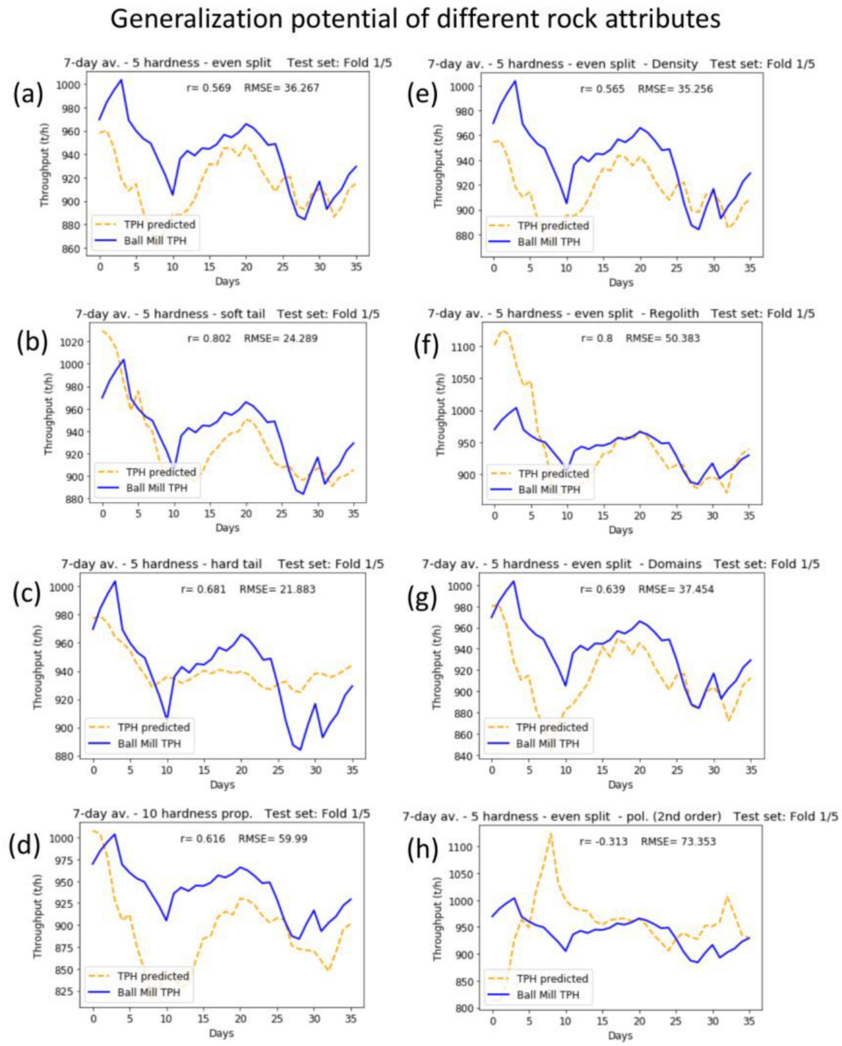


Figure 8: Generalization potential various of rock attributes testing on 20% unseen test data

notable correlation between the degree of weathering and ball mill throughput (Table 4). However, the generalization capability of weathering proportions is ambiguous, leading to a large overestimation of throughput in the first five days of the test set and increased RMSE of 50.38t/h (Figure 8f). The generalization capabilities of higher splits into ten hardness proportions (Figure 8d) and polynomial terms (Figure 8h) are poor. Overfitting is clearly visible for ten hardness proportions, which did not yet appear for previous leave-one-out cross-validation (Figure 7) because of the small ratio of testing vs. training data. The addition of second-order polynomials (Figure 8h) worsens the generalization potential further, resulting in even larger overfitting.

The approach presented in this article is general and allows the utilization of any rock attributes contributing to more accurate throughput prediction. In the present case study, the most robust predictions are expected from the utilization of five hardness proportions, as seen in Figure 8. This result forms the basis for the attributes used for short-term scheduling, presented next.

3.4 Short-term production scheduling

For weekly short-term production scheduling of the described mining complex, three months of planned production are used as input to optimization (3874 mining blocks, 12m $\times$ 12m $\times$ 12m). Various sets of geostatistical orebody simulations serve as input for production scheduling, displayed for Mining Area A in Figure 9. Uncertainty of metal grade (Au) is accounted for by twenty equally probable orebody scenarios. The required number of orebody scenarios for mine production scheduling has been examined by previous authors, showing that about 15 simulations provide proper stable schedules and forecasts (Albor Consuegra and Dimitrakopoulos, 2009; Montiel and Dimitrakopoulos, 2017). Note that each of the five hardness proportions forms an individual attribute for production scheduling and is represented by twenty orebody simulations each, as indicated in Figure 9. Economic parameters for the optimization are given in Table 5.

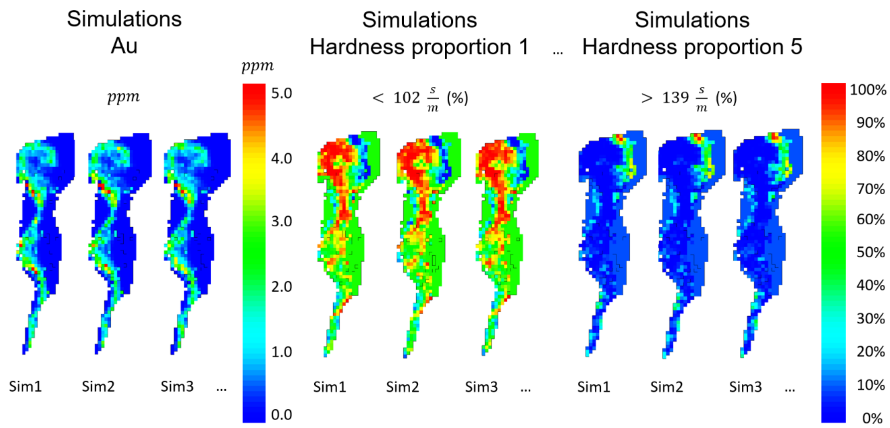


Figure 9: Orebody simulations of Mining Area A serving as input for short-term production scheduling using an integrated throughput prediction model

Table 5: Economic parameters used in this study

Parameter	Value
Periods	12 weeks
Gold price	1500$/oz
Selling cost	55$/oz
Mining cost – weathered rock	3.38$/t
Mining cost – fresh rock	3.58$/t
Processing cost	30.68$/t
Recovery	89.9%

A short-term production schedule is generated for the mining complex using the proposed method. Figure 10a shows a risk analysis of the tonnage that is expected to be processed in the ball mill. More specifically, Figure 10a shows the difference between the scheduled ore tonnage (blocks scheduled for production) and the tonnage that can be processed by the ball mill using the relevant geometallurgical attributes and the integrated throughput prediction model. The new stochastic constraints (Figure 10a, bottom) are directly linked to the objective function, as explained earlier.

It can be seen in Figure 10a that the expected difference between scheduled and processed ore (Figure 10a, solid black line) is small in all scheduled periods. Note that the use of a set of geological orebody scenarios (input to optimization) enables the evaluation of the risk of falling short of or surpassing required ore tonnage per period. There is a remaining risk involved of over and underfeeding the ball mill indicated by P10 and P90 risk profiles (Figure 10a dashed gray lines). However, the spread of the profile is kept well below 5 kt per period. The result indicates that the risk of not meeting scheduled ball mill production using the proposed method is low and can be controlled.

To evaluate the benefits of the new components for short-term production scheduling the case study is repeated without the use of the throughput prediction model. A conventional mill capacity constraint is used instead which limits the processed material per period based on a constant tonnage

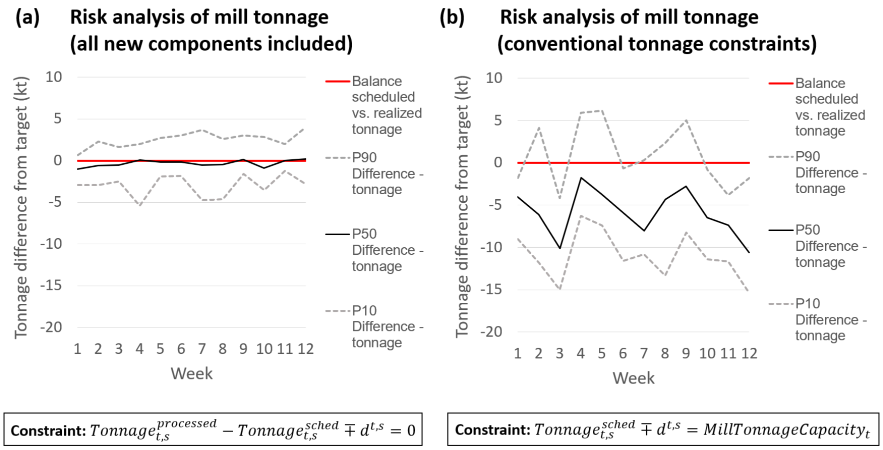


Figure 10: Risk analysis of stochastic production schedule including newly developed components (left) and using conventional tonnage constraints (right)

capacity. Figure 10b shows that the differences between scheduled tonnage and processed tonnage can be significant by using conventional capacity constraints. The largest deviations can be recognized in periods three and twelve. In both periods, an expected value of over10 kt of ore cannot be processed by the ball mill, which amounts to a deficit of 7% of the planned tonnage in each of these periods. This is indicated by the P50 value of the risk profile in Figure 10b (black line, solid). The risk evaluation also shows that there is a small probability of underfeeding the ball mill in some periods, indicated by the P90 risk profile exceeding the balance line (red). However, the expected cumulative shortfall of ore over the complete scheduling horizon (three months) accumulates to an expected value of 61 kt, which is equivalent to 3% of total production that cannot be achieved.

Mill capacity constraints that limit the amount of ore tonnage sent to the processing facility per period are standard practice, both in stochastic and deterministic production scheduling. However, these tonnage constraints assume a constant, average throughput of the milling and grinding circuit for every period and thus ignore the natural heterogeneity of rock attributes of processed ore, leading to highly variable throughput in the short-term, as it can be observed from daily and weekly production data displayed in Figure 6. The integrated prediction model is capable to assess the throughput and the associated risk profile to be expected from scheduled material sent to the processing plant. Figure 11 shows the ball mill throughput prediction for both schedules generated.

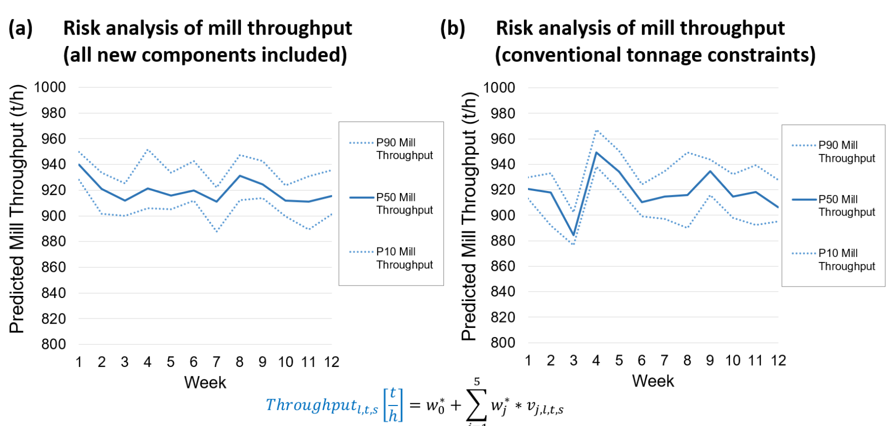


Figure 11: Risk analysis of expected ball mill throughput of production schedule using all new components (a) and using conventional mill capacity tonnage constraints (b)

While variability in throughput can be observed for both schedules, the variations in expected throughput from period to period are particularly large when conventional tonnage constraints are used (Figure 11b). There is a decrease of expected throughput from 920 t/h in period two to 884 t/h in period three (−4%). Considerable risks in production shortfall are the consequence, as seen in Figure 10b. These results emphasize the need for an integrated geometallurgical approach in mine production planning that creates short-term production schedules while integrating key metallurgical processes further downstream in mining complexes.

4 Conclusions

A new approach to integrate a geometallurgical throughput prediction model of the ball mill into short-term stochastic production scheduling in mining complexes has been presented. The throughput prediction model is constructed using production data generated in the mining complex, both in the mines (blasthole drilling, truck cycle data) and the processing plants (measured throughput of the operating ball mill). Penetration rates from blast-hole drilling are used to simulate the hardness and comminution behavior of the mineral reserve, replacing conventional comminution indices, which are typically costly and time consuming to obtain. The geostatistical simulation of hardness proportions presented herein avoids biases typically introduced by the change of support of non-additive geometallurgical properties. The additivity of the created hardness proportions enables the unbiased assessment of rock attributes of blended materials. Multiple regression is used to predict future throughput rates as a function of blended rock properties. Thus, the non-additive comminution behavior of blended materials is accounted for, overcoming the shortcomings of geometallurgical models that assess geometallurgical properties block by block. Finally, the integration of the throughput prediction model creates a much-needed link of adding non-additive geometallurgical properties to short-term production scheduling in mining complexes.

In the case study presented, the hardness proportions created show the highest correlation to ball mill throughput and result in the lowest prediction error among all tracked rock attributes. Regression analysis shows that throughput can be predicted using five hardness proportions with an RMSE of less than 30 t/h and a correlation coefficient of up to 0.8. Furthermore, it was demonstrated that splitting the harder and softer fractions of penetration rates differently leads to distinct prediction results, which could not be captured by the average-type approaches. By integrating the prediction model and the newly developed stochastic constraints to the optimization, the production schedule can achieve planned production with high certainty, better matching the scheduled materials with the predicted performance of the ball mill. Risk analysis of a weekly stochastic production schedule using conventional tonnage capacity constraints reveals that an expected value of more than 10 kt of ore cannot be processed by the ball mill in certain periods, which amounts to a deficit of 3% of planned tonnage over the three-month planning horizon.

Future work aims to further improve the prediction of non-additive geometallurgical properties by utilizing supervised learning models beyond multiple linear regression, which can account for non-linear relationships between predictor and response variables. For future prediction models, measured processing parameters, such as ball mill power and particle size distributions, should be considered given their direct influence on throughput rates. In addition, production data generated from other parts of the processing plant, such as mineral separation processes, should be considered to better integrate value-driving geometallurgical properties into the mine production scheduling.

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