Les Cahiers du GERAD ISSN: 0711–2440

Optimizing the mine production scheduling accounting for stockpiling and investment decisions under geological uncertainty

Z. Brika, M. Gamache, R. Dimitrakopoulos

G–2021–17

April 2021

La publication de ces rapports de recherche est rendue possible grâce au soutien de HEC Montréal, Polytechnique Montréal, Université McGill, Université du Québec à Montréal, ainsi que du Fonds de recherche du Québec – Nature et technologies.

Dépôt légal – Bibliothèque et Archives nationales du Québec, 2021
– Bibliothèque et Archives Canada, 2021

The publication of these research reports is made possible thanks to the support of HEC Montréal, Polytechnique Montréal, McGill University, Université du Québec à Montréal, as well as the Fonds de recherche du Québec – Nature et technologies.

Legal deposit – Bibliothèque et Archives nationales du Québec, 2021
– Library and Archives Canada, 2021

GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

Optimizing the mine production scheduling accounting for stockpiling and investment decisions under geological uncertainty

Zeyneb Brika [a, b]

Michel Gamache [a, b]

Roussos Dimitrakopoulos [a, c, d]

[a] *GERAD, Montréal (Qc), Canada, H3T 1J4*

[b] *Department of Mathematics and Industrial Engineering, Polytechnique Montréal, Montréal (Qc), Canada, H3c 3A7*

[c] *COSMO – Stochastic Mine Planning Laboratory, McGill University, Montréal (Qc), Canada, H3A 2A7*

[d] *Department of Mining and Materials Engineering, McGill University, Montréal (Qc), Canada, H3A 2A7*

michel.gamache@polymtl.ca
roussos.dimitrakopoulos@mcgill.ca

April 2021
Les Cahiers du GERAD
G–2021–17

Abstract : A new linear model is presented herein to optimize strategic production scheduling of an open pit mine with multiple processing streams while accounting for investment decisions under mineral supply uncertainty. The solution approach consists of first solving the linear relaxation using an extension of the Bienstock-Zuckerberg algorithm to the stochastic optimization. Then, a rounding heuristic based on the topological sorting is applied, followed by a parallel multi-neighborhood Tabu search. The proposed method is applied to a multi-product open pit mine deposit, with the possibility of investing in new shovels, trucks or crushers to increase related capacities. Numerical experiments show that the proposed method manages to solve the instance within an optimality gap less than 1.5% in reasonable time. Results also display an increased expected net present value of 6% compared to the formulation without investment options.

Keywords : Open pit scheduling, stochastic mathematical optimization, capital expenditures, parallelization

1 Introduction

The mineral value-chain represents the stages and transformations that the material goes through in a mining complex starting from the mines and passing through different processing streams before ending as a sellable product ready to be delivered to customers. All the components are interrelated, and the efficiency of the mining complex depends on the synergies between them. Thus, when it comes to determining an optimal strategic plan, they should be modeled and considered simultaneously. This approach has been referred to as a global optimization and its importance has been widely assessed in the technical literature (Hoerger et al., 1999 ; Pimentel et al., 2010 ; Bodon et al., 2011 ; Whittle, 2018). However, few researchers provide a mathematical framework to the problem while others try to optimize the different components independently, generating sub-optimal solutions for the mining complex as an entity. In the technical literature, the most common strategic mine planning problem considers one open pit mine discretized into a set of blocks, one processing stream, and one waste dump. The decision whether a block is considered as ore or waste material is commonly made *a priori*, which is referred to as a cut-off grade policy (Lane, 1988). The objective is to find a mining sequence that tells which blocks should be extracted and when so that the net present value (NPV) is maximized while respecting both slope and resource constraints. This problem is formulated as a knapsack problem (Kim, 1967 ; Johnson, 1969) and is considered NP-hard (Bienstock and Zuckerberg, 2010), making solving real-sized instances computationally challenging and beyond the limitation of current optimization methods. It goes without saying that incorporating additional operational constraints to make the model more realistic complicates matters further (Cullenbine et al., 2011). Over the decades, several solution approaches have been proposed. While some leverage the particular structure of the problem to use exact methods (Boland et al., 2009 ; Bienstock and Zuckerberg, 2010 ; Bley et al., 2010), others use heuristic and metaheuristic approaches (Ferland et al., 2007). Chicoisne et al. (2012), Lamghari et al. (2015) and Moreno et al. (2017) developed methods combining exact and approximate methods.

Despite the promising results of some of the methods mentioned above, they do not account for the uncertainties inherent to the problem and assume a perfect knowledge of the information, which leads to plans that fail to meet production forecasts (Baker and Giacomo, 1998). One of the main uncertainty sources is the geological supply. The uncertain nature of the geological supply stems from the fact that this information is inferred from limited drilling data and thus, cannot be fully known at the time scheduling is made. This uncertainty must be accounted for in the optimization process to generate reliable solutions that manage risk and maximize value (Ravenscroft, 1992 ; Dowd, 1994, 1997 ; Dimitrakopoulos et al., 2002). This issue was addressed by introducing a two-stage stochastic optimization model (Birge and Louveaux, 1997) that finds the optimal extraction sequence over a set of equally probable scenarios of the mineral deposit mined using spatial stochastic simulation methods (Ramazan and Dimitrakopoulos, 2005, 2013).

Later, several major extensions of this model that account not only for uncertainty, but also for optimizing the different components of the mining complex simultaneously in a single mathematical formulation have been proposed. Goodfellow and Dimitrakopoulos (2016, 2017) developed a stochastic integer nonlinear programming model that they solve with metaheuristics and that has the advantage of being easily adaptable to a large variety of applications ; this includes the application by Kumar and Dimitrakopoulos (2019), in which complex geo-metallurgical decisions are added to the destination policy and are applied to a large copper–gold mining complex. In a similar research vein, Montiel and Dimitrakopoulos (2015, 2017) suggested heuristic solution approaches to optimize the strategic production scheduling of a mining complex, incorporating operating alternatives for the transportation systems and processing modes. Zhang and Dimitrakopoulos (2018) find, by using a heuristic, near optimal strategic and tactical plans for a mining complex while considering uncertainties in both geological supply and the commodity prices. Considering the importance of moving towards the sustainable development of mineral resources, Rimélé et al. (2018) propose a new formulation that considers the management of mine waste and tailings. Before them, Zuckerberg et al. (2007), Ben-Awuah and

Askari-Nasab (2011) and Li et al. (2016) addressed the topic of in-pit waste disposal ; however, they did not consider geological uncertainty. Finally, Lamghari and Dimitrakopoulos (2019) propose a new hyper-heuristic that combines elements from reinforcement learning and Tabu search to solve complex stochastic scheduling problems arising in the mining industry.

Aiming to develop mathematical models that are closer to reality and more flexible, it is important to recall that each stage of the mineral value chain in a mining complex represents an added value over the previous one. It is therefore interesting to consider the different opportunities to invest at each of the major stages and thus, make the very most of mining. In this context, Ahmed et al. (2003) and Boland et al. (2008) have proposed a multi-period investment model for the capacity expansion problem under uncertain demand and cost parameters. However, a different solution is proposed for each scenario considered which is impractical. Goodfellow and Dimitrakopoulos (2016) extend the model introduced in Godoy and Dimitrakopoulos (2004) and propose a formulation allowing the optimizer to make new capital expenditure (CapEx) investments, such as purchasing new trucks or shovels, and thus, choose the optimal mining rate. Farmer (2016) also includes CapEx and mining capacity decisions as part of his model besides considering market uncertainty. However, the solution approach is a two-step process in which the extraction sequence and the downstream variables are optimized separately, leading to suboptimal solutions. Ajak et al. (2018) apply predictive data mining algorithms to forecast the occurrence of clay and thus, create possible real options at an operational level that bring flexibility to the management by dealing with clay uncertainty. Del Castillo and Dimitrakopoulos (2019) propose a new method that allows the mining complex to have more flexibility by dynamically offering over the life-of-the-mine (LOM) the possibility to make investments in strategic CapEx options, and thus, better adapt to new information.

This research proposes a tractable model that provides practical solutions for the long-term production scheduling of open pit mining complexes under multi-element geological uncertainty, where CapEx options are considered in the formulation. The uncertainty here is related to the content of the element(s) of interests and is illustrated by a set of equiprobable simulated geological scenarios of the deposit. The proposed model is an extension of the model presented in Brika et al. (2019) to include investment alternatives and is solved by using a decomposition method combined with heuristics.

The remainder of this paper is organized as follows. In Section 2, a mathematical model of open pit mine production scheduling optimization that accounts for capital investment options under geological uncertainty is presented. Then, in Section 3, the solving approach implemented along with the parallelization procedure is described, followed by an application in a multi-product iron ore deposit in Section 4. Conclusions follow in Section 5.

2 Mathematical formulation

To depict metal uncertainty, a finite set of equally probable scenarios are considered as input for the problem. Each scenario is a possible representation of the orebody's geological profile. Thus, a block could have different attributes depending on the scenario. The problem is then formulated as a two-stage stochastic integer programming model (Birge and Louveaux, 1997). In the first stage, the mining sequence and the blocks' destinations are determined and the different investment decisions as well. In the second stage, once the uncertainty is revealed, some violations could occur in terms of meeting production requirements. At this point, corrective actions can be taken as recourse generating some additional costs. Hence, the objective is to find a first-stage solution that would minimize the cost of the second-stage solution. In what follows, the notation and the mathematical model used to formulate the problem :

2.1 Notations

Sets	
$\mathcal{B} = \{1 \ldots B\}$	Set of blocks
$\mathcal{T} = \{1 \ldots T\}$	Set of time periods, which discretize the life of the mine
$\mathcal{M} = \{1 \ldots M\}$	Set of processing plants
$\mathcal{W} = \{1 \ldots W\}$	Set of waste dumps
$\mathfrak{D} = \mathcal{M} \cup \mathcal{W} = \{1 \ldots D\}$	Set of destinations including the waste dumps and the processing plants. By definition, the first M elements of $\mathfrak{D}$ will represent the processing plants $\{1 \ldots M\}$ and the remaining elements the waste dumps. Thus, $\mathfrak{D} = \{1 \ldots M, M+1, \ldots, M+W\}$.
$\mathcal{S} = \{1 \ldots S\}$	Set of possible scenarios of the orebody. Each scenario has the same probability of occurrence and represents a different simulation of the geological profile of each block.
$\mathcal{K} = \{1 \ldots K\}$	Set of investment options. The first M options correspond to investing in new crushers. Each crusher's type m is associated with a specific processing plant m. The two remaining options refer to investing in shovels and trucks. It is important to mention here that the fleets are considered homogeneous.
$\mathfrak{R} = \{1 \ldots R\}$	Set of ore properties (i.e. geological elements)
$\Gamma_b^+,\ \Gamma_b^-$	Set of immediate successors and predecessors of block b, respectively
Parameters	
$p_{b,d,t,s}$	Discounted profit obtained if block b is sent to destination d in period t under scenario s. If the destination is a processing plant, the profit is equal to the value of the metal content recovered of the block less the processing and selling costs. If the block is sent to the waste dump, it is equal to 0. Note here that the mining cost of the block is computed separately since processing and extraction could be done in different periods and thus, be subject to different economic discounts
$g_{b,s}^r$	Grade of block b for element r under scenario s
$Gmax_{m,t}^r$, $Gmin_{m,t}^r$	Expected maximum and minimum grades for resource r sent to processing plant m at period t
$mc_{b,t}$	Discounted cost of extracting a block b at period t
$rc_{b,m,t}$	Discounted cost of rehandling a block b from stockpiles to processing plant m at time t
cu_t and cl_t	Discounted unit costs of upper and lower deviations from the ore tonnage targets
cu_t^r and cl_t^r	Discounted unit costs of upper and lower deviations from $Gmax_{d,t}^r$ and $Gmin_{d,t}^r$.
Q_b	Tonnage of block b.
$d_{b,d}$	Approximative distance between a block b and destination d.
$ds_{b,m,t}$	Approximative distance between a block b and the stockpile associated with the mill m and the processing time t.
C_t^k	Discounted cost of investing in option k at period t.
κ^k	Unitary extra capacity obtained by the purchase of investment k.
τ^k, λ^k	Lead time and life of the equipment, respectively. The lead time is the time between the equipment is purchased and when it becomes operational.
N^k	Maximum number of equipment of type k to be purchased during the LOM.
$tonnage_t^k$	Total tonnage capacity initially available for equipment of type k for period t.
$time_t^k$	Time initially available for equipment of type k at period t.
η	Maximum number of trucks that can be loaded by one shovel.
h^k	Number of hours available per equipment of type k per year.
$vf,\ ve$	Speed of a truck when it is full and when it is empty, respectively.
$loadtime$	Unitary time to load one ton on the truck

2.1.1 Variables

In long-term mine planning models, two types of binary variables are used : the "at time t" variables and the "by time t" variables. Let $y_{b,t}$ be a binary variable that equals 1 if block b is extracted "at" or "by" time t. In the first case, a constraint $\sum_{t=1}^{T} y_{b,t} = 1$ must be used to ensure that block b is extracted once. In the second one, if $y_{b,t} = 1$ then $y_{b,t'} = 1\ \forall t^{'} = t+1,\ \ldots,\ T$ and the difference $y_{b,t} - y_{b,t-1}$ is used to know when the block was extracted. Using the "*by*" variables rather than the common "*at*" variables allows to represent both reserve and slope constraints as precedence constraints in a weighted directed graph. This graph will be explained in the following subsection. In the linear programming model presented in this paper, the *by* variables were used.

$y_{b,d,t} \in \{0,1\}$	Takes 1 if block b is completely extracted and sent to destination d by time t, 0 otherwise.
$z_{b,m,t_0,t_1} \in \{0,1\}$	Takes 1 if block b extracted and sent to stockpile by time t_0 and then sent to processing plant m at time t_1 with $(t_0 < t_1)$, 0 otherwise.
$w^k_{i,t} \in \{0,1\}$	Takes 1 if the element i of option k was purchased by period t, 0 otherwise.
$qu_{m,t,s},\ ql_{m,t,s} \ \in \mathbb{R}_+$	Continuous variables representing respectively the upper and lower deviation from ore tonnage production target at period t sent to processing plant m under scenario s.
$qu^r_{m,t.s},\ ql^r_{m,t.s} \ \in \mathbb{R}_+$	Continuous variables representing respectively the upper and lower deviation from production target for propriety r at period t sent to processing plant m under scenario s.

2.1.2 Precedence graph

A precedence graph, noted $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, is presented in Figure 1. Using the "*by*" variables rather than the common "*at*" variables allows to represent both reserve and slope constraints as precedence constraints in this weighted directed graph, in which each node refers to a decision variable $(y, zorw)$. Each constraint of type $\alpha \leq \beta$ is reflected in an arc (α, β), so if $\alpha = 1$, then $\beta = 1$ and all the variables associated with a node having node α as an ancestor will also be equal to 1. Artificial precedence constraints have been created between destinations d and between variables y and z. These precedence relationships do not affect the nature of the problem and ensure that each block is extracted at most once. The following paragraphs describe these precedence relationships.

If block a is extracted and sent to destination d (where d is either a processing plant or a waste dump, i.e. $d = 1, \dots, D$) by time t, then $y_{a,d,t} = 1$. Knowing that the constraint $y_{a,d,t} \leq y_{a,d+1,t}$ in the linear programming model presented in Section 2 will be associated with the arc $(y_{a,d,t}, y_{a,d+1,t})$, then the variable $y_{a,d+1,t} = 1$, and similarly $y_{a,d'}, t = 1\, \forall d' \in \{d+1, \dots D\}$. If block a was specifically extracted during period t, then there is a destination $d \in \mathfrak{D}$ such that $y_{a,d,t} - y_{a,d-1,t} = 1$ a and $y_{a,\alpha,t} - y_{a,\alpha-1,t} = 0\, \forall \alpha \neq d$.

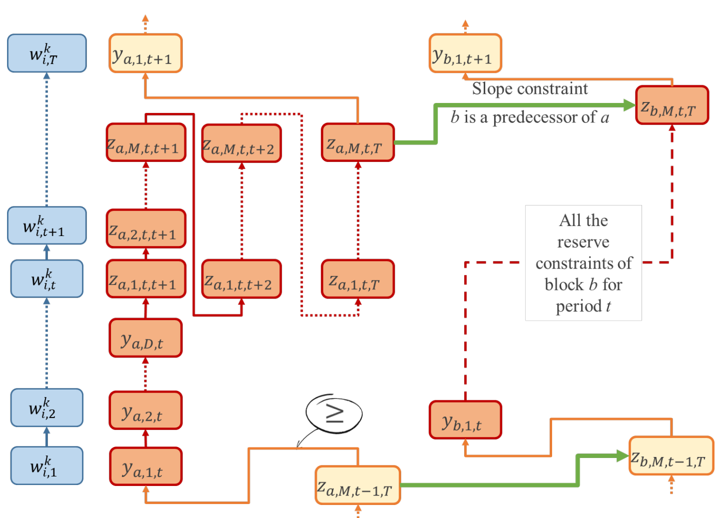

Figure 1 – Precedence constraints represented into a directed graph

If $y_{a,d,t} = 1$, then $y_{a,D,t} = 1$ and the arc $(y_{a,D,t}, z_{a,1,t,t+1})$ means that $z_{a,1,t,t+1} = 1$ and similarly $z_{a,d,t,t'} = 1\, \forall d \in \{2, .., M\}$ and $\forall t' \in \{t+1, .., T\}$. Moreover, $\forall d \in \{2, .., M\}$ and $\forall t' \in \{t+1, .., T\}$, $z_{a,d,t,t'} - z_{a,d-1,t,t'} = 0$. In other words, if block a has been extracted specifically during the period t and sent directly to a destination $d \in \mathfrak{D}$, then this block cannot be extracted and put on a stockpile at period t in order to be sent to the processing plant d at the period t'. Conversely, if $y_{a,D,t} = 0$ but $z_{a,M,t,T} = 1$, then there is a $d \in \{2, .., M\}$ and a $t' \in \{t+1, .., T\}$ such that $z_{a,d,t,t'} - z_{a,d-1,t,t'} = 1$.

In the precedence graph, there an arc $(z_{a,M,t,T}, z_{b,M,t,T})\, \forall a \in \Gamma_b^-$. These arcs represent the slope constraints that ensure that a block cannot be mined if not all its direct predecessors (i.e. the blocks directly above it in the geological block model) are already extracted. So, if $z_{a,M,t,T} = 1$, then each arc $(z_{a,M,t,T}, z_{b,M,t,T})$ such that $\forall a \in \Gamma_b^-$ will force $z_{b,M,t,T} = 1$, meaning that block b has been extract either at period t or before block a.

Finally, in the linear programming model, each unit of a type of equipment k that could be purchased during the life of the mine is identified by a particular index $i = 1, \ldots, N^k$. As each unit of equipment k can only be purchased once, the precedence constraints associated with the arcs $(w_{i,t}^k, w_{i,t-1}^k)$ will make it possible to check these constraints.

2.2 Mathematical model

2.2.1 Objective function

$$
\begin{aligned}
(\mathcal{P}) \quad \max \quad & \frac{1}{S}\sum_{b=1}^{B}\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{s=1}^{S}(p_{b,d,t,s} - mc_{b,t}) \times \check{y}_{b,d,t} && \text{(i)}\\
& + \left\{\frac{1}{S}\sum_{b=1}^{B}\sum_{m=1}^{M}\sum_{t_0=1}^{T-1}\sum_{t_1=t_0+1}^{T}\sum_{s=1}^{S}(p_{b,m,t_1,s} - mc_{b,t_0} - rc_{b,m,t_1}) \times \check{z}_{b,m,t_0,t_1}\right\} && \text{(ii)}\\
& - \frac{1}{S}\sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{r=1}^{R}\left(cu_t^r qu_{m,t.s}^r + cl_t^r ql_{m,t.s}^r\right) && \text{(iii)}\\
& - \frac{1}{S}\sum_{m=1}^{M}\sum_{t=1}^{T}\sum_{s=1}^{S}\left(cu_t qu_{m,t,s} + cl_t ql_{m,t,s}\right) && \text{(iv)}\\
& - \sum_{k=1}^{K}\sum_{i=1}^{N_k}\sum_{t=1}^{T} c_t^k \check{w}_{i,t}^k && \text{(v)}
\end{aligned}
$$

where :

$$
\check{y}_{b,d,t} = \begin{cases} y_{b,1,1}, & \forall b \in \mathcal{B}, d = 1, t = 1\\ y_{b,1,t} - z_{b,M,t-1,T}, & \forall b \in \mathcal{B}, d = 1, t = 2\ldots T\\ y_{b,d,t} - y_{b,d-1,t}, & \forall b \in \mathcal{B}, d = 2\ldots D, t = 1\ldots T \end{cases}
$$

$$
\check{z}_{b,m,t_0,t_1} = \begin{cases} z_{b,1,t_0,t_0+1} - y_{b,D,t_0}, & \forall b \in \mathcal{B}, m = 1, t_0 = 1\ldots T-1, t_1 = t_0 + 1\\ z_{b,1,t_0,t_1} - z_{b,M,t_0,t_1-1}, & \forall b \in \mathcal{B}, m = 1, t_0 = 2\ldots T-1, t_1 = t_0 + 2\ldots T\\ z_{b,m,t_0,t_1} - z_{b,m-1,t_0,t_1}, & \forall b \in \mathcal{B}, m = 2\ldots M, t_0 = 1\ldots T-1, t_1 = t_0 + 1\ldots T \end{cases}
$$

and :

$$
\check{w}_{i,t}^k = \begin{cases} w_{i,1}^k, & \forall k = 1\ldots K, i = 1\ldots N^k, t = 1\\ w_{i,t}^k - w_{i,t-1}^k, & \forall k = 1\ldots K, i = 1\ldots N^k, t = 2\ldots T \end{cases}
$$

The objective function presented herein is composed of five parts. Each part represents a different goal :

- **(i)** The first part aims to maximize the profits obtained by selling the blocks that were processed immediately after their extraction (i.e. without passing by a stockpile).
- **(ii)** The second part represents the profits by selling the blocks that were first extracted and then sent to a stockpile before being processed in later periods.
- **(iii)** The third part focuses on minimizing the deviations from the different production grade targets by penalizing them, a geological risk discounting factor (Ramazan and Dimitrakopoulos, 2013) is applied to the costs cu_t^r and cl_t^r to defer risk to later periods.

(iv) Similarly, the fourth part aims to minimize the deviations from the ore tonnage production targets and an economical discounting factor is also applied to the costs cu_t and cl_t.

(v) The fifth part ensures the minimization of capital expenditure costs.

2.2.2 Constraints

$$z_{b,M,t-1,T} \leq y_{b,1,t}, \qquad \forall b \in \mathcal{B}, t = 2 \dots T \tag{1a}$$

$$y_{b,d-1,t} \leq y_{b,d,t}, \qquad \forall b \in \mathcal{B}, d = 2 \dots D, t = 1 \dots T \tag{1b}$$

$$y_{b,D,t} \leq z_{b,1,t,t+1}, \qquad \forall b \in \mathcal{B}, t = 1 \dots T-1 \tag{1c}$$

$$z_{b,m-1,t_0,t_1} \leq z_{b,m,t_0,t_1}, \qquad \forall b \in \mathcal{B}, m = 2 \dots M, t_0 = 1 \dots T-1, t_1 = t_0+1 \dots T \tag{1d}$$

$$z_{b,M,t_0,t_1-1} \leq z_{b,1,t_0,t_1}, \qquad \forall b \in \mathcal{B}, m = 2 \dots M, t_0 = 1 \dots T-1, t_1 = t_0+2 \dots T \tag{1e}$$

Constraints (1) represent the reserve constraints, ensuring that a block cannot be mined and processed more than once during the lifetime of the mine.

$$w^k_{i,t-1} \leq w^k_{i,t}, \qquad \forall k = 1 \dots K, i = 1 \dots N^k, t = 2 \dots T \tag{2}$$

Simirarly, constraint (2) ensures that an equipment cannot be purchased more than once.

$$z_{a,M,t,T} \leq z_{b,M,t,T}, \qquad b \in \mathcal{B}, a \in \Gamma^-_b, t = 1 \dots T-1 \tag{3a}$$

$$y_{a,D,T} \leq y_{b,D,T}, \qquad b \in \mathcal{B}, a \in \Gamma^-_b \tag{3b}$$

Constraints (3) represent the slope constraints that ensure that a block cannot be mined if not all its direct predecessors (i.e. the blocks directly above it) are already extracted.

$$\sum_{b=1}^{B} Q_b \times \check{y}_{b,m,1} - qu_{m,1,s} \leq tonnage^m_t + \sum_{i=1}^{N_m} \sum_{p=\max(1,t-(\tau^m+\lambda^m))}^{t-\tau^m} \kappa^m \check{w}^m_{i,p},$$
$$\forall m = 1 \dots M, t = 1, s = 1 \dots S \tag{4a}$$

$$\sum_{b=1}^{B} Q_b \times (\check{y}_{b,m,t} + \sum_{t_0=1}^{t-1} \check{z}_{b,m,t_0,t}) - qu_{m,t} \leq tonnage^m_t + \sum_{i=1}^{N_m} \sum_{p=\max(1,t-(\tau^m+\lambda^m))}^{t-\tau^m} \kappa^m \check{w}^m_{i,p},$$
$$\forall m = 1 \dots M, t = 2 \dots T, s = 1 \dots S \tag{4b}$$

$$\sum_{b=1}^{B} Q_b \times \check{y}_{b,m,1} + ql_{m,1,s} \geq tonnage^m_t + \sum_{i=1}^{N_m} \sum_{p=t-(\tau^m+\lambda^m)}^{t-\tau^m} \kappa^m \check{w}^m_{i,p},$$
$$\forall m = 1 \dots M, t = 1, s = 1 \dots S \tag{5a}$$

$$\sum_{b=1}^{B} Q_b \times (\check{y}_{b,m,t} + \sum_{t_0=1}^{t-1} \check{z}_{b,m,t_0,t}) + ql_{m,t} \geq tonnage^m_t + + \sum_{i=1}^{N_m} \sum_{p=t-(\tau^m+\lambda^m)}^{t-\tau^m} \kappa^m \check{w}^m_{i,p},$$
$$\forall m = 1 \dots M, t = 2 \dots T, s = 1 \dots S \tag{5b}$$

Constraints (4) and (5) state that the total tonnage of the processed material plus or minus deviations must be within the upper and lower bound of the ore tonnage targets. If some crushers are bought, then a margin is added to the upper bound since the processing capacity is augmented.

$$\sum_{b=1}^{B} Q_b \times \left(g_{bs}^r - Gmax_{m,t}^r\right) \times \check{y}_{b,m,1} - qu_{m,1.s}^r \leq 0,$$
$$\forall m = 1 \dots M, t = 1, s = 1 \dots S, r = 1 \dots R \tag{6a}$$

$$\sum_{b=1}^{B} Q_b \times \left(g_{bs}^r - Gmax_{m,t}^r\right) \times \left(\check{y}_{b,m,t} + \sum_{t_0=1}^{t-1} \check{z}_{b,m,t_0,t}\right) - ql_{m,t,s} \leq 0$$
$$\forall m = 1 \dots M, t = 2 \dots T, s = 1 \dots S, r = 1 \dots R \tag{6b}$$

$$\sum_{b=1}^{B} Q_b \times \left(g_{bs}^r - Gmin_{m,t}^r\right) \times \check{y}_{b,m,1} + ql_{m,1.s}^r \geq 0,$$
$$\forall m = 1 \dots M, t = 1, s = 1 \dots S, r = 1 \dots R \tag{7a}$$

$$\sum_{b=1}^{B} Q_b \times \left(g_{bs}^r - Gmin_{m,t}^r\right) \times (\check{y}_{b,m,t} + \sum_{t_0=1}^{t-1} \check{z}_{b,m,t_0,t}) + ql_{m,t.s}^r \geq 0,$$
$$\forall m = 1 \dots M, t = 2 \dots T, s = 1 \dots S, r = 1 \dots R \tag{7b}$$

Constraints (6) and (7) ensure that the resulted grades of the extracted material plus or minus deviations are within the upper and lower ranges.

$$\sum_{b=1}^{B} Q_b \times \left(\sum_{d=1}^{D} \check{y}_{b,d,t} + \sum_{p=t+1}^{T} \sum_{m=1}^{M} \check{z}_{b,m,t,p}\right) \leq tonnage_t^{shovel}$$
$$+ \sum_{i=1}^{N_{shovel}} \sum_{p=\max(1,t-(\tau^{shovel}+\lambda^{shovel}))}^{t-\tau^{shovel}} \kappa^{shovel} \check{w}_{i,p}^{shovel}, \qquad \forall t = 1 \dots T-1 \tag{8a}$$

$$\sum_{b=1}^{B} Q_b \times \sum_{d=1}^{D} \check{y}_{b,d,t} \leq tonnage_t^{shovel}$$
$$+ \sum_{i=1}^{N_{shovel}} \sum_{p=\max(1,t-(\tau^{shovel}+\lambda^{shovel}))}^{t-\tau^{shovel}} \kappa^{shovel} \check{w}_{i,p}^{shovel}, \qquad t = T \tag{8b}$$

Constraints (8) ensure that the tonnage of extracted material does not exceed the mining capacity. If some shovels are bought, then the mining capacity is augmented.

$$\sum_{b=1}^{B} \sum_{d=1}^{D} Q_b \times \left[\frac{1}{\kappa^{truck}} \times d_{b,d} \times \left(\frac{1}{vf} + \frac{1}{ve}\right) + loadtime\right] \times \check{y}_{b,d,t}$$
$$\leq time_t^{truck} + h^{truck} \times \sum_{i=1}^{N_{truck}} \sum_{p=\max(1,t-(\tau^{truck}+\lambda^{truck}))}^{t-\tau^{truck}} \check{w}_i^{truck}, p, \quad t = T \tag{9a}$$

$$\sum_{b=1}^{B} \sum_{d=1}^{D} Q_b \times \left[\frac{1}{\kappa^{truck}} \times d_{b,d} \times \left(\frac{1}{vf} + \frac{1}{ve}\right) + loadtime\right] \times \check{y}_{b,d,t}$$
$$+ \sum_{b=1}^{B} \left(\sum_{p=t+1}^{T} \sum_{m=1}^{M} Q_b \times \left[\frac{1}{\kappa^{truck}} \times ds_{b,m,p} \times \left(\frac{1}{vf} + \frac{1}{ve}\right) + loadtime\right] \times \check{z}_{b,m,t,p}\right)$$
$$\leq time_t^{truck} + h^{truck} \times \sum_{i=1}^{N_{truck}} \sum_{p=\max(1,t-(\tau^{truck}+\lambda^{truck}))}^{t-\tau^{truck}} \check{w}_{i,p}^{truck}, \quad \forall t = 1 \dots T-1 \tag{9b}$$

Constraints (9) ensure that the time required to load the blocks and send them to their respective destinations during a period does not exceed the trucks' total availability time.

3 Solution approach

The solution approach presented herein is a three-step method. Section 3.1 illustrates the first step which consists in solving optimally the linear relaxation of the problem (LP) described in the previous section using the Bienstock-Zuckerberg algorithm. The second step described in Section 3.2 is a greedy heuristic that takes the fractional solution provided by Step 1 as input and rounds it, while ensuring that no hard constraints are violated. The last and third step presented in Section 3.3, consists in applying a Tabu search heuristic to improve the quality of this new integer solution. To explore the different neighborhoods, parallelization technics are used to speed up the computational time.

3.1 Solving the linear relaxation

If all the side constraints were relaxed and only the integrity and precedence constraints were maintained, the problem ($\mathcal{P}$) could be treated as a maximum closure problem applied to the previous graph and can be solved in polynomial time (Hochbaum, 2008). The Bienstock-Zuckerberg algorithm (Bienstock and Zuckerberg, 2010) takes advantage of this particular structure to solve efficiently the problem. The main idea behind the algorithm is to use the Lagrangian relaxation and iteratively replace, until reaching convergence, the Lagrangian multipliers that penalize the violation of the relaxed constraints in the objective function. The algorithm alternates between two steps. At a given iteration, first, a maximum closure problem is solved, and a new partition (nodes in the closure and those out of the closure) is obtained and then intersected with the previous ones to get a new partition. Second, the resulting new partition allows to reduce the number of variables in the LP by forcing variables whose nodes belong to the same group in the partition to be equal and then, be treated as a single variable while preserving the individual properties over all scenarios. Once the LP is solved, the dual variables of the side constraints aim to update the Lagrangian multipliers. The same process is repeated until it converges, and an optimal solution for the LP is then obtained (for more details, see Munoz et al., 2018).

3.2 Finding a feasible integer solution

Starting from the fractional solution obtained at the first step described in Section 3.1, the second step consists in finding a feasible integer solution with a relatively good quality. In what follows, the whole process is detailed :

Step 0 : A weighted *Toposorting* is applied to obtain a sequence of extraction. It consists in sorting the blocks in a sequence $\{b_1, b_2, \dots, b_n\}$ with respect to a weight vector w where (b_i, b_{i+1}) satisfies either $\{b_i$ predecessor of $b_{i+1}\}$ or $\{w_i < w_{i+1}\}$. The weight vector w can be calculated as the "expected" time of extraction for block b based on the optimal solution of the LP obtained at the first step. For further details, the reader is referred to Brika et al. (2018) since a similar sorting is used.

Step 1 : The fractional solution is simply rounded to the closest integer values. Thus, a new integer solution is obtained. However, it is important to mention that for the equipment variables, the two closest integer values are tested. Since the process is very fast (few seconds) and the number of combinations remains limited, the different options can be tested and only the best one is retained. Once the rounding is done, the solution obtained does respect the slope constraints but not necessarily the resources constraints. For each block, the extraction period, the processing period and the destination obtained by rounding the fractional are set as default values.

Step 2 : Starting with period 0 up to the last period, as long as at least one hard constraint is violated, the extraction of some blocks is delayed while favouring the postponement of those that arrive later in the sorted sequence as shown in Figure 2.

Step 3 : A second round is done to improve the quality of current solution. Following the same Toposort ordering, the blocks are moved one by one from a period to another and a destination to another so that the Objective function's value is maximized while respecting all the hard constraints.

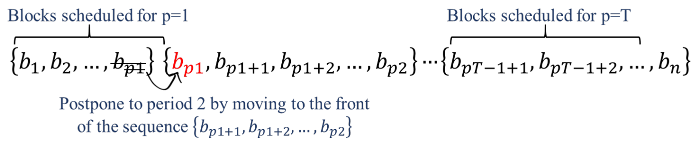

Figure 2 – Repairing the solution by postponing the extraction of some blocks

3.3 Improving the solution

The purpose of the last step is to improve the quality of solution obtained in Section 3.2 by applying a Multi-Neighborhood Tabu Search (MNTS) for each period. At a given iteration p, one neighborhood is formed by postponing to $p+1$ the extraction of blocks originally scheduled in period p. Another consists in pushing forward to p the extraction of blocks originally scheduled in period $p+1$. The same changes could be done for the processing time. In addition to changing the periods, each neighborhood is expanded by the possibility of changing the destinations. It goes without saying that only neighbors that respect the hard constraints (i.e. precedence constraints and equipment availability) are retained.

Creating the different neighbourhoods is a computationally intensive task and doing it for each period only makes things worse. A parallelization strategy is then implemented to reduce the running time spent searching the neighbourhoods. It consists in launching multiple threads simultaneously that run independently with a specific neighbourhood, and then synchronized to share the best moves found so far. To ensure that the search step in each thread is perfectly independent, the neighborhoods are built so that a given period cannot be affected in more than one thread at once. More formally, each Tabu Search thread (TST) is associated with two successive periods ($p,\ p+1$). The first thread $TST_{(1,2)}$ will make moves between periods (1,2), $TST_{(3,4)}$ between periods (3,4) and so on until the last two periods. Also, at $TST_{(p,p+1)}$, a destination change cannot be done if not both extraction and processing time occur between p and $p+1$. Annexe 1 summarizes all the neighbours that can be explored inside a given thread $TST_{(p,p+1)}$. Each TST is initialized with the same solution and runs independently a Tabu search exploring the associated neighborhoods until reaching a stop criterion. Once all the TST have finished their running, all the best moves are applied to the initial solution. Then, to explore more solutions, a second round should be done with threads $TST_{(2,3)}$, $TST_{(4,5)}$, etc. The general framework of the algorithm is shown in Figure 3.

4 Numerical results

4.1 Implementation

The aforementioned model is coded in C++ and the tests were run on an Intel (R) Core (TM) i5-8250U CPU computer (1.60 GHz) with 8 GB of RAM operating under Windows 10 environment. Recall that the first step consists in solving the linear relaxation to optimality. This step alternates between solving a linear problem with CPLEX 12.7 using default settings and solving the maximum closure problem with the pseudo flow algorithm introduced by Hochbaum, 2008. The second step which is a rounding heuristic does not need any parameter. Finally, as a third step, the Tabu search needs three parameters to be set. The size of the Tabu list is fixed at 0.6N where N is the approximative number of blocks that can be moved at each iteration (i.e. the blocks located in the inner and outer borders for a given period). The second parameter is the stop criterion. It represents the maximum number of successive non-improving iterations and was fixed to N. These two values were defined based on preliminary tests. The third parameter represents the number of threads to use. Different values were tested in order to assess the usefulness of the parallelization. However, it is worth mentioning that having more threads than available cores in the computer used is meaningless.

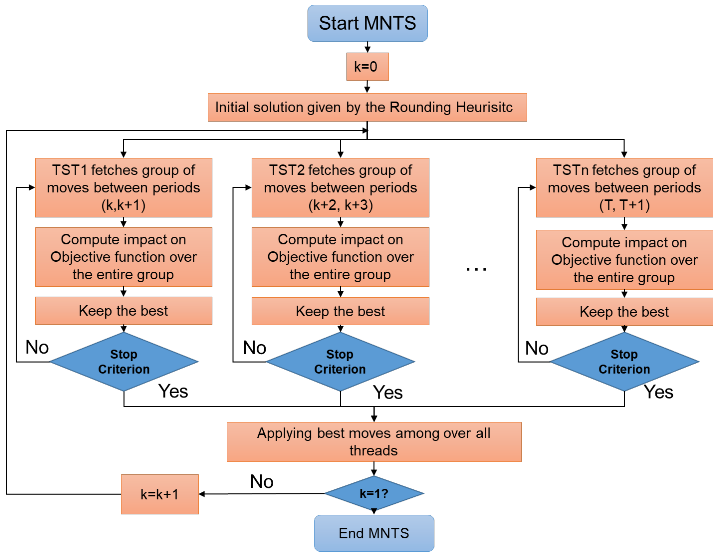

Figure 3 – The multi-neighborhood Tabu search process

4.2 Data

The data set consists of one multi-product iron open pit mine deposit of 33168 blocks (25 x 25 x 2 meters). Two elements of interest are considered : Iron (Fe) and Silica (SiO2). The mining complex considers multiple destinations including stockpiles, waste dumps and different processing facilities. No cut-off is used, the optimizer decides where to send each block. The grade requirements were given by the companies as well as the initial mining capacity and the ore tonnage targets. To depict the *in-situ* variability, 10 equiprobable geological simulations were used for each instance (Boucher and Dimitrakopoulos, 2009 ; Godoy, 2003). Regarding the penalties, they were fixed by trial-and-error. Table 1 shows the mining and economic parameters of the case study and Table 2 illustrates the different production targets and requirements for each processor.

Table 1 – Mining and economical parameters of the Iron mine

Parameters	Value
Mining Cost ($/t)	5.0
Stockpile reclaim cost ($/t)	0.5
Initial mining capacity (Mt)	16
Discount rate (%)	10
Ore tonnage penalty cost ($/t)	25.0
SiO2 penalty cost ($/t)	10.0
Fe penalty cost ($/t)	10.0
Life time of the mine (years)	5

Table 2 – Products' requirements

Parameters	Processor 1	Processor 2
Final product price ($/t)	30.00	26.00
Processing cost ($/t)	6.00	5.00
Fe grade target (%)	[57.7, 59.6]	[56.0, 58.7]
SiO2 grade target (%)	[4.6, 5.2]	[4.1, 5.0]
Recovery rate (%)	100	

4.3 Investment options

Integrating the CapEx alternatives adds some flexibility to the model by offering the possibility to increase both extraction and processing capacities. Four CapEx options are considered in this case study :

1. Invest in new trucks and thus, increase extraction capacity.
2. Invest in new shovels and thus, increasing extraction capacity if linked to available trucks.
3. Invest on a new crusher that allows increasing the processing capacity at the first processor.
4. Invest on a new crusher that allows increasing the processing capacity at the second processor.

Information about each of the CAPEX options are summarized in Table 3 :

Table 3 – Information about each investment option

	Truck	Shovel	Crusher of Processor 1	Crusher of Processor 2
Purchase price ($)	480,000.00	3,200,000.00	4,500,000.00	4,500,000.00
Life of equipment (years)	6	10	15	15
Lead time (years)	1	1	2	2
Maximum to purchase	5	2	1	1
Initial capacity	22,000 machine-hour / year	16Mt / year	8 ; 6 ; 6 ; 3.3 ; 4.2 Mt for year 1, 2, ... 5 respectively	6 ; 4 ; 4 ; 5 ; 2.5 Mt for year 1, 2, ... 5 respectively
Extra capacity per unit purchased	180t per loading, 7,300 machine-hour / year	10 Mt	1 Mt	1 Mt
Speed (km/h)	40 empty /30 full	–	–	–

4.4 Results

4.4.1 Running times and optimality gaps

Table 4 shows results of running the proposed methodology on the case study presented earlier. Solving the linear relaxation with the Bienstock-Zuckerberg (BZ) algorithm converges quickly and reaches optimality after 6 minutes. Then, the fractional solution is first rounded to the closest integers (SR) and then this solution is repaired and improved by applying the rounding heuristic (RH) and the tabu search heuristic (MNTS). Both heuristics are fast (few seconds). The first one provides an integer solution within a gap of 64.5% when compared to the optimal solution of the linear relaxation. The second rounding heuristic improves it to reach a gap of 2%. After applying the Tabu search, the final gap is around 1.5%. The running time of the latter depends on the number of threads used in the parallelization process.

Table 4 – Running times and optimality gaps

BZ		SR			RH			MNTS		
Time	Gap(%)	Time	Gap (%)	Diff (%)	Time	Gap (%)	Diff (%)	Time	Gap (%)	Diff (%)
6 min	0.0	4 s	64.5	–	5 s	2.0	62.5	6 – 16 min	1.5	0.5

Table 4 indicates that the second rounding heuristic has the biggest impact in terms of improving the solution quality. By contrast, the Tabu search seems to have a very limited impact while being the most time consuming. Its poor performance can be explained by two reasons. First, the solution provided by the rounding step has already a very small gap. It becomes easier to be stuck in local optima. Second, to not exclude potential good solutions, the neighbourhood at each iteration is kept very large. Exploring all neighbours takes time.

4.4.2 Comparing running time with different threads

The gains obtained by adding the parallelization to the Tabu search algorithm in terms of running times are illustrated in Figure 4 :

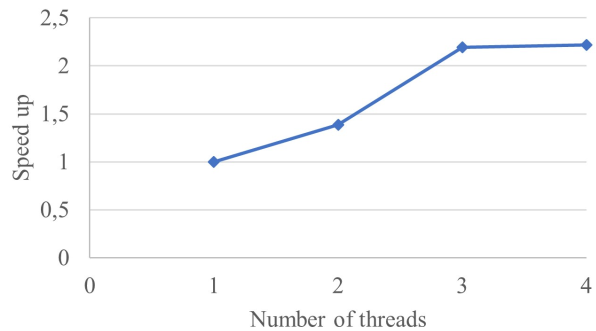

Figure 4 – Parallelization effects

As noted in Section 4.1, the used machine has only 4 cores. For this reason, the maximum number of threads tested is four. Furthermore, since only five years of production are considered, and each thread affects a distinct pair of years, the maximum number of pairs to evaluate simultaneously is three. Thus, considering more than three threads becomes is meaningless. Overall, for the proposed implementation, the search is executed twice faster using three threads than using a single threading.

4.4.3 Effect of adding investment options

The solver decided to invest in two new crushers at the first period : one for each processor. The effects of this acquisition in terms of ore tonnage can be observed in Figure 5 and Figure 6 starting from the third year considering there is a lead time of two years. Overall, at both processors, the ore tonnage production increases by $16-40\%$ when compared with the basic model that does not consider CapEx alternatives in the optimization process.

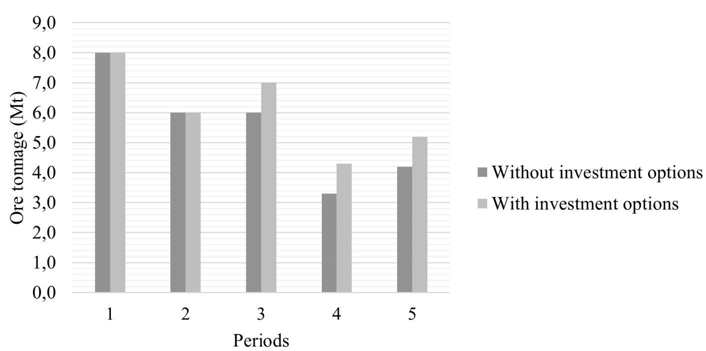

Figure 5 – Ore tonnage production at processor 1

The benefits are also reflected in the cumulative NPV of the project as it can be seen in Figure 7. The two curves represent the cumulative NPVs of the models with and without CapEx alternatives. It can be observed that during the first three years, the curve of the model considering the options is below the basic model's one. This is explained by the cost of the two crushers' acquisition. This difference is then offset by a higher production. Adding flexibility to the model ultimately resulted in a 6% higher NPV.

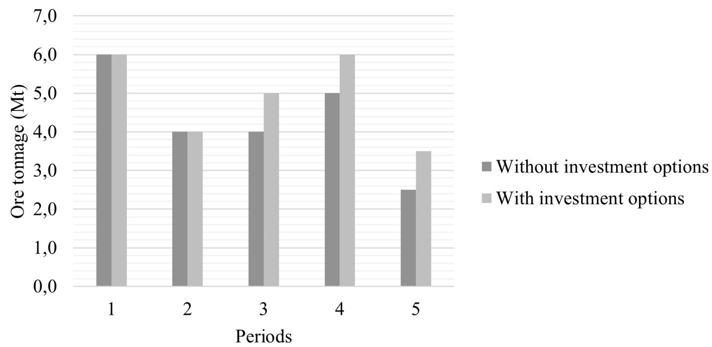

Figure 6 – Ore tonnage production at processor 2

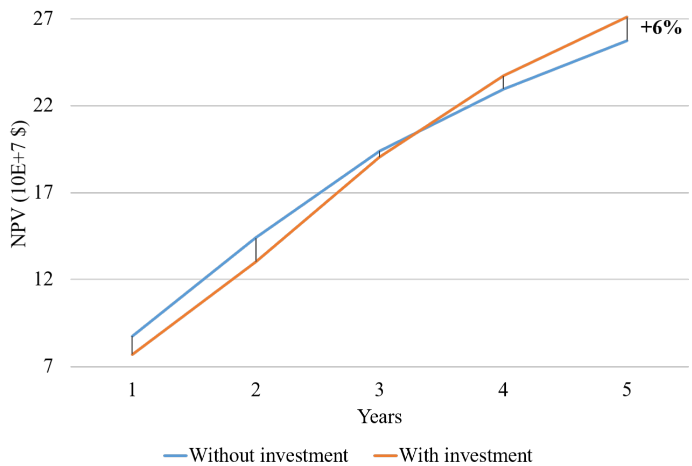

Figure 7 – Cumulative NPV with and without CapEx alternatives

5 Conclusions

In this paper, a new stochastic model is developed to integrate flexibility into the long-term scheduling optimization process of open pit mine deposits. This flexibility is reflected in the possibility of increasing the extraction capacity through purchasing new trucks and shovels and/or the processing capacity by acquiring new crushers. The solution approach proposed by Brika et al. (2019) has been extended to consider CapEx alternatives. Among other things, repairing steps had been added to the rounding heuristic to handle the hard constraints induced by including the equipment. A parallelized multi-neighbourhood Tabu search algorithm has also been implemented to efficiently improve the solution provided by the rounding heuristic.

A case study is presented at an iron open pit mining complex comprised of one pit with two possible processing destinations, waste dump and stockpiles. Results show that the algorithm performs well. An integer solution within an optimality gap of 1.5% is obtained in reasonable time ($12 - 22$ minutes in total). The parallelization also managed to reduce the running time proportionally to the number of threads used until a certain limit. In terms of mining operations, results show that investing in a secondary crusher in the first year for both processors represents an overall increase in NPV of over 6%, as compared to the basic model without investment alternatives.

In a future study, the proposed method should be tested on bigger instances to better assess its robustness. In addition, since the Tabu search's performance was debatable, it would be interesting to refine the way the neighborhoods are built to reduce their sizes, and thus speed up the process. Not fixing the CapEx alternatives at this step could also be considered in order to observe more significant effects on the solution value. Finally, future work will also focus on extending the formulation to include heterogeneous fleets.

References

Ahmed, S., King, A., Parija, G. : A multi-stage stochastic integer programming approach for capacity expansion under uncertainty. Journal of Global Optimization, 26 : 3–24. DOI : 10.1023/A :1023062915106 (2003).

Ajak, A.D, Lilford, E., Topal, E. : Application of predictive data mining to create mine plan flexibility in the face of geological uncertainty. Resources Policy, 55 (2018) : 62–79. DOI : 10.1016/J.RESOURPOL.2017.10.016 (2018).

Akaike, A., Dagdelen, K. : A strategic production scheduling method for an open pit mine. Proceedings, 28th International APCOM symposium ; Computer applications in the minerals industries ; Golden, CO, 729–738 (1999).

Albor, F., R. Dimitrakopoulos, R. : Stochastic mine design optimization based on simulated annealing : Pit limits, production schedules, multiple orebody scenarios and sensitivity analysis, Mining Technology, 118(2), 79–90. DOI : 10.1179/037178409X12541250836860 (2009).

Albor, F., Dimitrakopoulos, R., 2010. Algorithmic approach to pushback design based on stochastic programming : method, application and comparisons. Mining Technology 119(2), 88–101 (2010).

Baker, C.K., Giacomo, S.M. : Resource and reserves : their uses and abuses by the equity markets. Ore reserves and Finance : A Joint Seminar between Australasian Institute of Mining and Metallurgy and ASX, Sydney (1998).

Ben-Awuah, E., Askari-Nasab, H. : Oil sands mine planning and waste management using mixed integer goal programming, International Journal of Mining, Reclamation and Environment, 25(3) : 226–247, DOI : 10.1080/17480930.2010.549656 (2011).

Benndorf, J., Dimitrakopoulos, R. : Stochastic long-term production scheduling of iron ore deposits : integrating joint multi-element geological uncertainty. Journal of Mining Science, 49(1), 68–81. DOI : 10.1134/S1062739149 010097 (2013).

Bienstock, D., Zuckerberg, M. : A new LP algorithm for precedence constrained production scheduling. Optimization Online (2009).

Bienstock, D., Zuckerberg, M. : Solving LP relaxations of large-scale precedence constrained problems. Proceedings from the 14th conference on Integer Programming and Combinatorial Optimization (IPCO). Lecture Notes in Computer Science 6080, 1–14. DOI : 10.1007/978-3-642-13036-6_1 (2010).

Birge, J.R., Louveaux, F. : Introduction to Stochastic Programming. Springer (1997).

Bley, A., Boland, N., Fricke, C., Froyland, G. : A strengthened formulation and cutting planes for the open pit mine production scheduling problem. Computers & Operations Research, 37(9) : 1641–1647 (2010).

Bley, A., Boland, N., Froyland, G., and Zuckerberg, M. : Solving mixed integer nonlinear programming problems for mine production planning with stockpiling. Journal of Mining Science, 47 (3), 338–359. http://www.optimization-online.org/DB_HTML/2012/11/3674.html (2012).

Bodon, P., Fricke, C., Sandeman, T., Stanford, C. : Modeling the mining supply chain from mine to port : A combined optimization and simulation approach. Journal of Mining Science, 47(2) : 202–211 (2011).

Boland, N., Dimitrescu, I., Froyland, G. : A multistage stochastic programming approach to open pit mine production scheduling with uncertain geology. Optimization Online, http://www.optimization-online.org/DB_FILE/2008/10/2123.pdf. Visited 19 July 2019 (2008).

Boland, N., Dumitrescu, I., Froyland, G., Gleixner, A. : LP-based disaggregation approaches to solving the open pit mining production scheduling problem with block processing selectivity. Computers and Operations Research, 36(4), 1064–1089 (2009).

Boucher, A., Dimitrakopoulos, R. : Block simulation of multiple correlated variables. Mathematical Geosciences 41(2), 215–237 (2009).

Brika, Z., Gamache, M., Dimitrakopoulos, R. : Multi-Product Mine Scheduling Optimization under Multi-Element Geological Uncertainty. Cahier du GERAD. G–2018–72 (2018).

Brika, Z., Gamache, M., Dimitrakopoulos, R. : Solving the mixed-integer linear programming problem for mine production scheduling with stockpiling under multi-element geological uncertainty. Cahier du GERAD. G–2019–05 (2019).

Caccetta, L., Hill, S. P. : An application of branch and cut to open pit mine scheduling. Journal of Global Optimization, 27(2–3), 349–365 (2003).

Chicoisne, R., Espinoza, D., Goycoolea, M., Moreno, E., Rubio, E. : A new algorithm for the open-pit mine production scheduling problem. Operations Research, 60(3), 517–528 (2012).

Cullenbine, C., Wood, R.K., Newman, A. : A sliding time window heuristic for open pit mine block sequencing. Optim Lett, 5(3), 365–377. DOI : 10.1007/s11590-011-0306-2 (2011).

Dagdelen, K., Johnson, T.B. : Optimum open pit mine production scheduling by lagrangian parameterization. Proceedings of 19th International APCOM Symposium, Littleton, CO, 127–142 (1986).

Del Castillo, M.F., Dimitrakopoulos, R. : Dynamically optimizing the strategic plan of mining complexes under supply uncertainty. Resources Policy, 60 : 83–93 (2019).

Denby, B., Schofield, D. : Open pit design and scheduling by use of genetic algorithms. Transactions of the Institutions of Mining and Metallurgy l. 103, A21–A26 (1994).

Dimitrakopoulos, R. : Stochastic optimization for strategic mine planning : A decade of developments. Journal of Mining Science, 84, 138–150 (2011).

Dimitrakopoulos, R., Farrelly, C. T., Godoy, M. C. : Moving forward from traditional optimization : grade uncertainty and risk effects in open pit design. Mining Technology, 111 (2002).

Dimitrakopoulos, R., Ramazan, S. : Stochastic integer programming for optimizing long-term production schedules of open pit mines : methods, application and value of stochastic solutions. Mining. Technololy, 117(4), 155–160. DOI : 10.1179/174328609X417279 (2008).

Dowd, P. : Risk assessment in reserve estimation and open-pit planning. Transactions of the Institution of Mining and Metallurgy (Section A : Mining Industry), 103, A148–A154 (1994).

Dowd, P. : Risk in minerals projects : analysis, perception and management. Mining Technology, 106 (1997).

Farmer, I. : Stochastic mining supply chain optimization : A study of integrated capacity decisions and pushback design under uncertainty. M.Sc. thesis, McGill University, Canada (2016).

Ferland, J.A., Amaya, J., Djuimo, M.S. : Application of a particle swarm algorithm to the capacitated open pit mining problem. In, S. Mukhopadhyay, G. Sen Gupta (Eds.), Autonomous Robots and Agents, Springer-Verlag. 127–133 (2007).

Godoy, M. : The effective management of geological risk. Ph.D. thesis, University of Queensland, Australia (2003).

Godoy, M., Dimitrakopoulos, R. : Managing risk and waste mining in long-term production scheduling. SME Transactions. 316, 43–50 (2004).

Goodfellow, R., Dimitrakopoulos, R. : Global optimization of open pit mining complexes with uncertainty. Applied Soft Computing. 40, 292–304. DOI : 10.1016/j.asoc.2015.11.038 (2016).

Goodfellow, R., Dimitrakopoulos, R. : Simultaneous stochastic optimization of mining complexes and mineral value chains. Mathematical Geosciences, 49(3), 341–360 (2017).

Hochbaum, D. : A new-old algorithm for minimum-cut and maximum-flow in closure graphs. Networks, 37(4) : 171–193. DOI : 10.1002/net.1012, MR 1837196 (2001).

Hochbaum, D. : The pseudoflow algorithm : a new algorithm for the maximum flow problem. Operations Research 58, 992–1009 (2008).

Hoerger, S., Seymour, F., Hoffman, L. : Mine planning at Newmont's Nevada operations. Mining Engineering, 51(10), 26–30 (1999).

Johnson, T.B. : Optimum open pit mine production scheduling. Ph.D. thesis, University of California, Berkeley (1968).

Johnson, T.B. : Improving Returns from Mine Products Through Use of Operation Research Techniques. US Bureau of Mines, Report of investigations 7230, Washington, DC (1969).

Kim, Y.C. : Mathematical Programming Analysis of Mine Planning Problems. Ph.D. Thesis, The Pennsylvania State University, University Park, PA (1967).

Kumar, A., Dimitrakopoulos, R. : Application of simultaneous stochastic optimization with geometallurgical decisions at a copper-gold mining complex. Mining Technology. DOI : 10.1080/25726668.2019.1575053 (2019).

Lamghari, A., Dimitrakopoulos, R. : A diversified Tabu search approach for the open pit mine production scheduling problem with metal uncertainty. European Journal of Operational Research, 222(3) 642–652 (2012).

Lamghari, A., Dimitrakopoulos, R., Ferland, J. : A hybrid method based on linear programming and variable neighborhood descent for scheduling production in open-pit mines. Journal of Global Optimization 63(3) : 555–582. DOI : 10.1007/s10898-014-0185-z (2015).

Lamghari, A., Dimitrakopoulos, R. : Progressive hedging applied as a metaheuristic to schedule production in open-pit mines accounting for reserve uncertainty. European Journal of Operational Research, 253 : 843–85 (2016).

Lamghari, A., Dimitrakopoulos, R. : Hyper-heuristic approaches for strategic mine planning under uncertainty. Computers & Operations Research. Available online 17 November 2018. DOI : 10.1016/j.cor.2018.11.010 (2019).

Lane, K. : The economic definition of ore : cut-off grades in theory and practice. Mining Journal Books, 0900117451 (1988).

Li, Y., Topal, E., Ramazan, S. : Optimising the long-term mine waste management and truck schedule in a large-scale open pit mine. Mining Technology, 125(1) : 35–46. DOI : 10.1080/14749009.2015.1107343 (2016).

Marcotte, D., Caron, J. : Ultimate open pit stochastic optimization. Computers & Geosciences, 51, 238–246 (2013).

Menabde, M., Froyland, G., Stone, P., Yeates, G. : Mining schedule optimization for conditionally simulated orebodies, in : R. Dimitrakopoulos (Ed.), Orebody Modelling and Strategic Mine Planning : Uncertainty and Risk Management Models, AusIMM Spectrum Series, 2007, 379–384 (2007).

MineMax. MineMax planner. Retrieved from : http://www.minemax.com/. Visited in : 07.09.18 (2016).

Mintec, Inc. Minesight. Retrieved from : http://www.minesight.com/. Visited in 05.09.18 (2013).

Montiel, L., Dimitrakopoulos, R. : Optimizing mining complexes with multiple processing and transportation alternatives : An uncertainty-based approach. European Journal of Operational Research, 247(1), 166-178. DOI : 10.1016/j.ejor.2015.05.002 (2015).

Montiel, L., Dimitrakopoulos, R. : A heuristic approach for the stochastic optimization of mine production schedules. J. Heuristics, 23 : 397–415 (2017).

Moreno, E., Rezakhah, M., Newman, A., Ferreira, F. : Linear models for stockpiling in open-pit mine production scheduling problems. European Journal of Operational Research, 260(1), 212–221. DOI : 10.1016/j.ejor.2016.12.014 (2017).

Munoz, G., Espinoza, D., Goycoolea, M., Moreno, E., Queyranne, M., Rivera, O. : A study of the Bienstock-Zuckerberg algorithm. Computational Optimization and Applications, 69(2) :501–534. DOI : 10.1007/s10589-017-9946-1 (2018).

Pimentel, B., Mateus, G., Almeida, F. : Mathematical models for optimizing the global mining supply chain. Intelligent Systems in Operations : Methods, Models and Applications in the Supply Chain, Business Science Reference, 133–163 (2010).

Ramazan, S. : A new pushback design algorithm in open pit mining. MSc thesis, Department of Mining Engineering, Colorado School of Mines, Golden, Colorado, USA (1996).

Ramazan, S. : The new fundamental tree algorithm for production scheduling of open pit mines. European Journal of Operational Research, 177(2), 1153–1166 (2007).

Ramazan, S., Dimitrakopoulos, R. : Recent applications of operations research in open pit mining. Trans. Soc. Min. Metall. Explor. 316, 73–78 (2004).

Ramazan, S., Dimitrakopoulos, R. : Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation. In, Orebody Modelling and Strategic Mine Planning, Spectrum Series 14, Australasian Institute of Mining and Metallurgy (AusIMM), 359–365 (2005).

Ramazan, S., Dimitrakopoulos, R. : Production scheduling with uncertain supply : a new solution to the open pit mining problem. Optimization and Engineering, 14(2), 361–380. DOI : 10.1007/s11081-012-9186-2 (2013).

Rezakhah, M., Newman, A. : Open pit mine planning with degradation due to stockpiling, Computers and Operations Research. DOI : 10.1016/j.cor.2018.11.009 (2018).

Rimélé, A., Gamache, M., Dimitrakopoulos, R. : A stochastic optimization method with in-pit waste and tailings disposal for open pit life-of-mine production planning. Resources Policy, 57, 112–121. DOI : 10.1016/j.resourpol.2018.02.006 (2018).

Shishvan, M.S., Sattarvand, J. : Long-term production planning of open pit mines by ant colony optimization. European Journal of Operational Research, 240(3), 825–836 (2015).

Smith, M. L. : Optimizing inventory stockpiles and mine production : an application of separable and goal programming to phosphate mining using AMPL/CPLEX. CIM Bulletin, 92(1030), 61–64 (1999).

Smith, M. L., Wicks, S. J. : Medium-term production scheduling of the Lumwana mining complex. Interfaces, Special Issue : Operations Research in Mining, 44(2), 176–194 (2014).

Tabesh, M., Askari-Nasab, H. : Two-stage clustering algorithm for block aggregation in open pit mines. Mining Technology, 120, 158–169 (2011).

Tabesh, M., Askari-Nasab, H., Peroni, R. : A comprehensive approach to strategic open pit mine planning with stockpile consideration. In Proceedings of the Thirty-seventh international symposium on applications of computers and operations research in mineral industry, 326–332. Society for Mining, Metallurgy and Exploration (2015).

Tachefine, B. : Méthode d'optimisation pour la planification de la production dans une mine à ciel ouvert. Ph.D. thesis, Ecole polytechnique de Montréal, QC (1997).

Topal, E. : Advanced underground mine scheduling using mixed integer programming. Ph.D. thesis, Colorado School of Mines, Golden, CO (2003).

Whittle, J. : Beyond optimization in open pit design. Proc Canadian Conf on Computer Applications in the Mineral Industries, Rotterdam, pp 331–337 (1988).

Whittle Programming. Gemcom Whittle User Help v4.4, Section : Stockpiles – General discussion (2010).

Whittle, J. : The Global Optimiser Works - What Next ? In Dimitrakopoulos R. (ed) Advances in Applied Strategic Mine Planning, Springer International Publishing, Cham, 31–37. DOI : 10.1007/978-3-319-69320-0_3 (2018).

Yarmuch, J., Ortiz, J. : A novel approach to estimate the gap between the middle- and short-term plans. In Proceedings, 35th APCOM symposium 2011 - Application of Computers and Operations Research in the Minerals Industry, 24–30 September 2011, University of Wollongong, WA, Australia. (2011).

Zhang J., Dimitrakopoulos R. : Optimising a mineral supply chain under uncertainty with long-term sales contracts. In : Dimitrakopoulos R. (ed) Advances in Applied Strategic Mine Planning. Springer, Cham., 787–800 DOI : 10.1007/978-3-319-69320-0_45 (2018).

Zuckerberg, M., Stone, P., Pasyar, R., Mader, E. : Joint ore extraction and in-pit dumping optimization. Orebody Modelling and Strategic Mine Planning : Uncertainty and Risk Management Models (2nd ed), Australasian Institute of Mining and Metallurgy (AusIMM), 137–140 (2007).