Les Cahiers du GERAD

A model of early-stage finance

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Abstract: In recent years, we have witnessed a mini-revolution around early-stage financing. In some places, like the UK, more money is raised on Equity Crowdfunding (ECF) platforms than by angels and VCs combined. In this paper we provide a simple theoretical framework for early-stage financing. In particular, we examine how the decision of the entrepreneurs to opt for equity crowdfunding instead of funding with angel investors – who add value but take more equity – affects the failure rate of a startup and its future value. We show that for startups where there may be little a priori support, because, for example, the entrepreneurs are inexperienced or because the idea is very different from anything out there, a successful ECF campaign can result in a significant increase in value. We specify the general conditions that determine whether entrepreneurs are better off opting for angel investments or for an ECF campaign as a function of various parameters, including the wisdom (or lack thereof) of the crowds.

Keywords: Equity crowdfunding, angel investor, early-stage finance, entrepreneurial finance

1   Introduction

Much has been said in recent years about Fintech and its impact (or lack thereof) on banking and finance. However, one area where it has undoubtedly had a significant impact is early-stage financing. In particular, equity crowdfunding (hereinafter ECF), which started in 2012 in the UK, now accounts for more than 40% of early-stage financing in the UK; globally in 2018, ECF accounted for about $1.51 billion in investments, up from $1.15 billion in 2017 (Ziegler et al. 2020). An ECF campaign consists in a startup company posting a short pitch (in almost all cases a video) and bios of the founders. Investors then choose whether and how much to invest. Thousands of investors invest in each single campaign, in return for equity in the company. As with other types of crowdfunding, entrepreneurs set a campaign goal that must be reached in a predetermined period (typically 60 days). If there are enough pledges, the campaign succeeds, the startup receives the investment and the investors, via the platform, become shareholders. If the campaign goal is not reached by the deadline, the campaign fails.

The UK is widely acknowledged as the most developed equity crowdfunding market, largely because its Financial Conduct Authority (the relevant regulator) adopted a laissez- faire approach in the early days of the industry. But ECFs are now popular and growing fast in many parts of the world, including the US, especially since the introduction of the JOBS act.

This paper was written during the lockdown of 2020. While it is too early to tell what the long term implications of Covid will be on early stage finance, it seems likely that this trend will continue to grow in the coming years. Investors, big and small, are more comfortable with making investment decisions online, and the frequent travelling associated with the "road show" required to raise money from angel investors seems less and less attractive. In fact, anecdotal evidence from the top two UK platforms (SEEDRS and CrowdCube, with a market share of over 80% between them) shows that after the first two weeks of lockdown, where investors were more cautious, more campaigns were launched and more money raised than ever before.

As one would expect, the growth of ECF platforms had a particularly large impact on angel investing. In the early days, ECFs were seen as an alternative to seeking angel investors. But a lot has changed since 2012 and angel investors now account for an increasing percentage of ECF investments (see Estrin et al, 2018, and Wang et al, 2019) where we track this upward trend in co-investment patterns. Angel investors are thought of as bringing additional value to the firm, in other words, more than just money. The fact that they now participate in most ECF campaigns suggests a fundamental change in this thinking. The goal of this paper is to provide a theoretical framework with which it is possible to study and understand the different incentives in early-stage equity investing, and how these might differ between ECFs and angel investing. In particular, our framework shows the interplay between the additional value that can be gained from the wisdom of the crowd and the additional value angel investors may bring.

Despite the growing academic interest in crowdfunding, there has been little in the way of theory. For reward-based crowdfunding, the seminal contributions of Strausz (2017) and Ellman & Hurkens (2019) are notable exceptions: the general idea conveyed in these papers is that, faced with uncertain demand for her or his product, the entrepreneur can benefit by launching presales on a crowdfunding site, learning about the shape of the demand curve for the product and then optimizing the price or quantity of production. Furthermore, Strausz (2017) shows that these presales on a reward-based crowdfunding site are optimal in terms of mechanism design.

There are some important differences between reward and equity crowdfunding: investors may back a startup whose products they are not interested in consuming or purchasing, or a startup that is not consumer-oriented at all, because they believe it has the potential to succeed and will therefore provide them with a good return on their investment. Still, reward and equity crowdfunding share one element of the wisdom of the crowd, which our model highlights.

It is worth noting that compared to reward-based platforms, the investment amounts on ECF platforms are significantly larger. An average campaign on Kickstarter, a leading reward-based platform, raises US$23K, compared to about £292K (and thus well over 10 times as much) on SEEDRS (the UK's second largest ECF platform). The existing literature on equity crowdfunding focuses mostly on investor behaviour. One

central question concerns the wisdom of the crowd: is there herd behaviour, and is that behaviour rational? Estrin & Khavul (2015) examine this question with UK data from CrowdCube, whereas Astebro et al. (2017) approach this question using SEEDRS data. More recently, Hellmann et al. (2019) considers the strategies of the entrepreneurs, asking how gender and campaign choices affect campaign outcomes.

In the first part of the paper, we consider only the ECF route. We set up a three-period model. In the first stage, entrepreneurs set their goal and valuation. If the campaign is successful, then we look at the value of the startup post campaign. Finally, the third stage is what we call the scale-up stage — if the startup is still alive by that point, then it is ready for series A funding or an early exit.

We begin the analysis by providing a detailed framework for how entrepreneurs set a campaign goal and a valuation. Unlike raising money from angels or VCs where everything is negotiable, entrepreneurs setting up their ECF campaign can get it wrong, for example not just by asking for too much or too little for their plan, but also by getting the valuation wrong in a way that will discourage the crowd from investing in the project.

We derive what should be the optimal campaign goal and valuation, considering how wise the crowd is, the form of the "penalty" for setting your goal and valuation wrong, and finally properties of the project and the team.

We then look at what happens post campaign: a successful campaign always means an increased valuation for the company (because it now has real rather than potential value) and this can be very significant. Our model shows that the less likely the startup was to succeed in the first place, e.g. because the idea is completely new or the founders unknown, the greater the post-campaign jump in value is. This can be very significant, up to a factor of 10 in some circumstances. Hence, the public validation of this new idea has substantial economic value. Agritech business Hectare (formerly SellMyLivestock) is a good example. The unknown founders had an idea that was very different from what anyone had ever seen (a marketplace for selling livestock). They launched an ECF campaign in 2016 to raise 370K GBP. The company is now valued at 18 million GBP, and at the time of writing, had not yet had its series A. Interestingly, our model predicts the post-campaign jump to be stronger the wiser the crowd is.

In the second part of our model we consider the entrepreneurs' choice of using an ECF in the first instance, by considering its advantages and disadvantages compared with taking money from an angel investor instead. An angel investor (or a group of angels acting together) will typically want a higher share of the company compared with the crowd but could potentially add value.

Our model then identifies conditions under which entrepreneurs are better off with ECF, depending of course on parameters such as how much value is added by the angel and how much more equity she or he demands.

Our main result here that for the "left of centre" ideas, entrepreneurs are ALWAYS better off with the crowd because if the crowd funds them, the startup will experience a significant increase in value and survival rate that no angel investor can provide. This is consistent with initial data from the UK, as in Beauhurst (2020).

For all other startups, we find a general expression showing when the entrepreneur is better off with an angel. Interestingly, for almost all reasonable parameters, the angel can only charge very little or entrepreneurs will prefer the potential upside from a successful ECF campaign. This last result is consistent with most empirical studies (see Wang et al., 2019, and references within) which find a significant decline in angel investing since ECFs began, and at the same time, growing participation of angel investors in ECF campaigns.

The rest of the paper is organized as follows: Section 2 presents the ECF model, and describes entrepreneurs' and investors' decisions and their impact on ECF failure probability and startup failure probability. Section 3 introduces the angels as another source of funding. Section 4 concludes.

2 Equity crowdfunding model

We consider a three-period model. At time $t = 0$, the entrepreneur starts an equity crowdfunding (ECF) campaign that ends at $T_1 = 1/6$ year (60 days). If successful, the project goes on and the future project value at time T_2 (in years) is modelled by a random variable $V_{T_2}^{\text{ECF}}$ with expectation

$$\mu_V = \text{E}^{\mathbb{P}} \left[V_{T_2}^{\text{ECF}} \right].$$

T_2 is the "next stage" of the start-up. We have in mind the time when seed capital runs out (also known as "the end of the runway"). At that point, the startup may be dead, or ready for more funding (typically series A), or even an early exit. The point being that the startup, if still alive, is now worth more than it was at the seed-funding stage because of what it has done during this stage.

     ECF campaign   Scale-up stage

    0   T_1 T_2

Figure 1: Time scale.

2.1 At time $t = 0$

The initial project value is the expected discounted future value

$$V_0 = \text{E}_0^{\mathbb{P}} \left[(1 + R)^{-T_2} V_{T_2}^{\text{ECF}} \right] = (1 + R)^{-T_2} \mu_V, \tag{1}$$

where R denotes the (annualized) rate of return on investment. There is uncertainty regarding the future value of the project $V_{T_2}^{\text{ECF}}$. A typical approach is to consider different scenarios (optimistic, realistic and pessimistic) and granting them a probability. Another approach is to specify a distribution for the random variable $V_{T_2}^{\text{ECF}}$ and characterize its moments. In all cases, the expected future value $\mu_V = \text{E}^{\mathbb{P}} \left[V_{T_2}^{\text{ECF}} \right]$ of the project is based on a model intended to be an approximation of reality. Thus V_0 is not known with certainty: entrepreneurs and investors have a noisy estimate of this quantity.

Definition 1 (Entrepreneur's Decisions) *The campaign goal is x which has to be reached before T_1. The proportion α_{ECF} of the company the crowd gets in returns for x.*[1]

2.2 At time $T_1 = 60$ days

The campaign fails if the total funding X_{T_1} raised during the campaign is below the campaign goal x, that is $X_{T_1} < x$. The crowd reacts to the specificity of a project as well as to the entrepreneur's decisions. Consequently, X_{T_1} is a random variable with expectation m and standard deviation s where these first two moments are some functions of the fundraising campaign characteristics and the entrepreneur's decisions.

The general point of all kinds of crowdfunding is that there is "wisdom in the crowds," a term originally coined by a journalist in Plymouth in 1906 observing how good the crowd was in estimating the weight of an ox. In more modern times, Iyer et al. (2015) show that crowdfunders can be as good, if not better, than experts in predicting credit scores and other economic features.

We now model just how wise (or not) the crowd is, in the following way:

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[1]Virtually all ECF platforms allow entrepreneurs to raise more money once the goal is reached. This is in return for further equity, the exact rules differing between platforms. For example, in SEEDRS, any money raised is raised at the same equity rate, so for example if the entrepreneurs ask for 100K for 10% but end up raising 200K, then they have to give up 20% of the equity.

Definition 2 (Crowd's Characteristic) *Depending on who the investors are, the crowd can be more or less knowledgeable about the project value. This is captured by the uncertainty (or noise) parameter σ that affects both the expected total funding and its variability: the smaller the noise parameter σ is, the better informed the crowd is about the fair project value V_0.*

Key to our model is that entrepreneurs must get their startup valuation (roughly) right or they will not be funded. For most startups, there is actually quite a lot to go by (using what is known as analogues, similar businesses in similar industries). In fact, the platforms themselves provide entrepreneurs with a lot of information on how to go about doing that.[2]

We now provide a formal definition of how this "penalty" works: the crowd is far less likely to invest if the valuation is "way off". We note though that if the amount being asked for is reasonable and the valuation is actually slightly below what is "right," then this presents itself as a good opportunity to buyers, i.e. the crowd, and therefore could lead to a small boost in demand for that company.

Definition 3 (Crowd's decision) *Knowing that the entrepreneur offers participation at the level of α_{ECF}, the fair campaign goal should be*

$$x_0 = \alpha_{\mathrm{ECF}} V_0$$

while the entrepreneur is asking for an amount of x. Since the investors have only a noisy appreciation of the project's initial value V_0, the same applies to the fair campaign goal.

Penalty. *The crowd penalizes the ECF campaigns for which the gap between x and x_0 is too large. To capture the uncertainty about x_0, the distance $x - x_0$ is normalized by the noise parameter σ. This penalty has no effect when $x = x_0$ and decreases as the magnitude of*

$$z = \frac{x - x_0}{\sigma} \tag{2}$$

increases.

"Good buy" premium. *The crowd perceives a campaign goal x slightly below the fair value x_0 as an opportunity to buy a project at a low cost. More precisely, let $\varepsilon > 0$ be a small positive number. A campaign goal that belongs to the interval $(x_0 - \sigma\varepsilon, x_0)$ increases the likelihood of an ECF campaign's success.*

These two effects are captured via the function

$$h(z, \varepsilon) = \exp\left(-\frac{1}{\varepsilon} z (z + \varepsilon)\right) \tag{3}$$

that has no effect whenever $x = x_0$ or $x = x_0 - \sigma\varepsilon$ is greater than one whenever the campaign goal x belongs to the interval $(x_0 - \sigma\varepsilon, x_0)$ and decreases at an exponential rate as the distance z increases. Figure 2 illustrates the effect of the parameter ε.
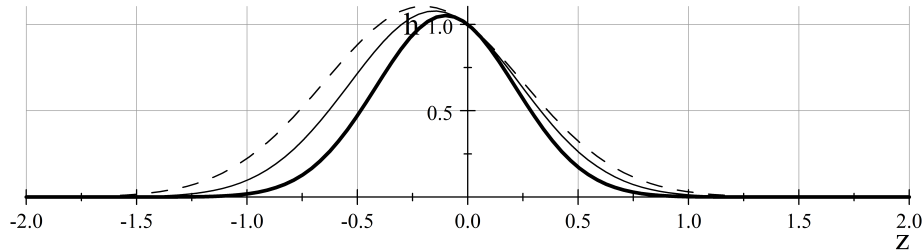
Figure 2: Penalty function. This figure illustrates the penalty (3) with respect to the distance to the fair value (2). The case where the campaign goal x is set to its fair value x_0 corresponds to $z = 0$. The good buy premium is located to the left of the origin. The parameter $\varepsilon \in \{20\%$ (thick), 30% (continuous), 40% (dashed)$\}$ controls for the "good buy" premium location.

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[2]https://www.seedrs.com/learn/help/how-do-i-value-my-business-2

The empirical literature on ECF shows clearly that campaigns led by experienced entrepreneurs (e.g. who led a previous start-up to a successful exit) had a higher overall success rate (see Vulkan et al. (2016), Estrin & Khavul (2016), and Hellmann et al. (2019)). We model this as follows:

Definition 4 (Crowd's decision) *The expected funding is also affected by the quality of the project/ entrepreneurial team which is captured by the parameter ρ. An entrepreneurial team that has experienced some success in the past and submitted a promising project could spark investors' enthusiasm and see its funding facilitated. This situation is captured by letting $\rho > 1$. Other projects involving an entrepreneurial team with little or no experience or unusual projects may be less attractive to investors. These cases are represented with $\rho < 1$.*

Axiom 1 *The funding X_{T_1} is a random variable centred at*

$$m\left(x, x_0, \sigma, \rho, \varepsilon\right) = x_0 \rho h \left(\frac{x - x_0}{\sigma}, \varepsilon\right) \tag{4}$$

with a standard deviation

$$s\left(x, x_0, \sigma\right) = \sigma x h \left(\frac{x - x_0}{\sigma}, \varepsilon\right). \tag{5}$$

Figure 3 presents the first two moments of the total funding with respect to the campaign-goal-to-fair-value ratio x/x_0. As there is wisdom in the crowd, the expected total funding base scenario is the unknown fair value x_0. The project type and/or some particularities associated with the entrepreneurial team affect this base case through the parameter ρ. Therefore, if the entrepreneur sets the campaign goal to its fair value, $x = x_0$, the expected funding is $x_0 \rho$. The continuous thick lines in the left panel of Figure 3 correspond to the expected funding of a standard project with $\rho = 1$. The unusual projects with $\rho < 1$, represented with dashed lines, have lower expected funding.

The crowd can be more, or less, wise. We model this using the parameter σ. The green curves in Figure 3 present the case of a wise crowd: the expected funding is more substantial when x is in the neighbourhood of its fair value x_0 and the variability (right panel) is low. The black curves represent an uninformed crowd where the total funding can depart from its fair value without a significant penalty, and where the variability is more important.

In all cases, the "good buy" premium sets the maximum expected funding at a campaign goal x slightly to the left of its fair value x_0, that is the ratio x/x_0 slightly below one. The expected funding decreases rapidly as x/x_0 departs from one, that is, an entrepreneur who sets the campaign goal too far from its fair value will see its expected funding greatly reduced.
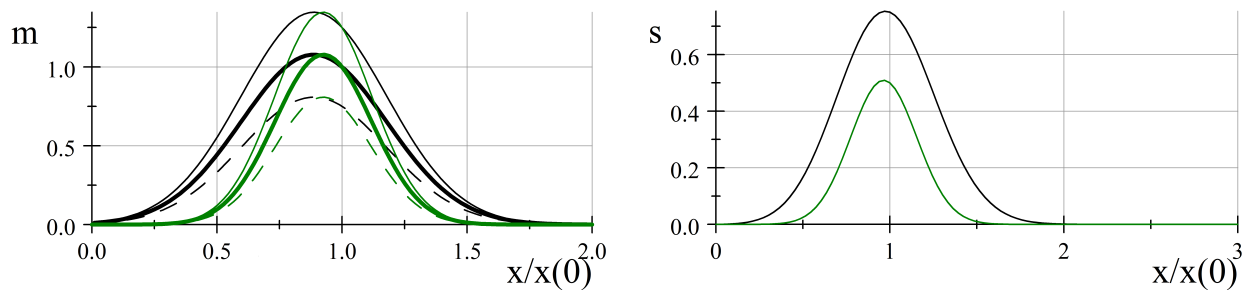
Figure 3: Funding expected value and standard deviation. The right panel presents the expected funding (4) and the left panel is the total funding standard deviation (5). The parameters are $x_0 = 1$, $\sigma \in \{\frac{3}{4}$ (black), $\frac{1}{2}$ (green)$\}$; $\rho \in \{0.75$ (dashed), 1 (thick), 1.25 (continuous)$\}$ and $\varepsilon = 30\%$.

Let

$$Z_{T_1} = \frac{X_{T_1} - m\left(x, x_0, \sigma, \rho, \varepsilon\right)}{s\left(x, x_0, \sigma\right)}$$

be the standardized funding. The ECF campaign failure probability is

$$\mathbb{P}\left[X_{T_1} < x\right] = \mathbb{P}\left[Z_{T_1} < \frac{x - m\left(x, x_0, \sigma, \rho, \varepsilon\right)}{s\left(x, x_0, \sigma\right)}\right] = F_{Z_{T_1}}\left(\frac{x - m\left(x, x_0, \sigma, \rho, \varepsilon\right)}{s\left(x, x_0, \sigma\right)}\right) \tag{6}$$

where $F_{Z_{T_1}}$ is the cumulative distribution function of Z_{T_1}.

Figure 4 illustrates the failure probability of an ECF campaign, seen as a function of the campaign goal over the fair value x/x_0, for various scenarios. The obvious observation is that the projects identified with $\rho < 1$ have larger probabilities of campaign failure since, for any fixed level of uncertainty, the curves are one above the other, the dashed lines being above. When there is wisdom in the crowd (green lines), the ECF failure probability is high as soon as the campaign goal x departs from its fair value.

Figure 4: Failure probability of an ECF campaign. This figure presents the ECF campaign failure probability (6). The parameters are $x_0 = 1$, $\sigma \in \{\frac{3}{4}$ (black), $\frac{1}{2}$ (green)$\}$; $\rho \in \{0.75$ (dashed), 1 (thick), 1.25 (continuous)$\}$ and $\varepsilon = 30\%$. For this specific illustration, we assume that the total funding X_{T_1} is normally distributed. The shape can vary with other distributions, but their interrelation remains.
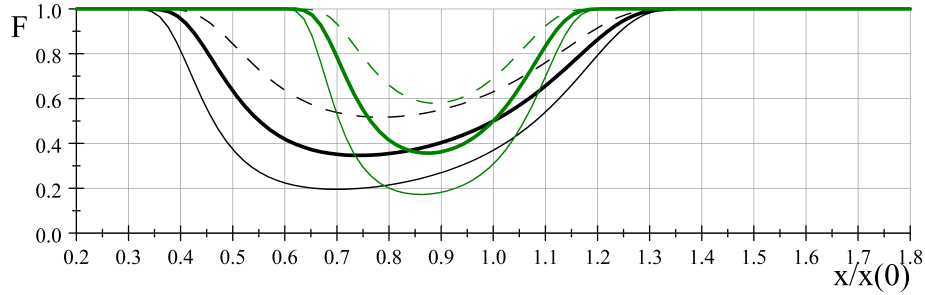
2.3   At time T_2

In this section, we study the expected future value of the project conditional on the success of the ECF campaign, and measure its impact on the startup failure probability. We show that for projects with high a priori risk of failure, the additional value from a successful ECF campaign can be significant.

2.3.1   Expected project value at time T_2

In this section, the ECF campaign failure probability (6) is denoted F.

Theorem 1 *The project's expected future value, given that the campaign is successful, is*

$$\mathrm{E}^{\mathbb{P}}\left[V_{T_2}^{ECF}|X_{T_1} \geq x\right] = \frac{\mu_V}{1 - F}. \tag{7}$$

The proof is in Appendix 4. Intuitively, the ECF campaign allows crowd information to come to light. Thus, the highly uncertain projects which succeed during the crowdfunding campaign are potentially better than initially evaluated. Consider the ratio

$$\xi\left(x\right) = \frac{\mathrm{E}^{\mathbb{P}}\left[V_{T_2}^{\mathrm{ECF}}|X_{T_1} \geq x\right]}{\mathrm{E}^{\mathbb{P}}\left[V_{T_2}^{\mathrm{ECF}}\right]} = \frac{1}{1 - F} \tag{8}$$

describing by how much the expected future value of the project increases when the ECF campaign is successful.

Figure 5 presents the ratio of conditional expected project value at T_2 knowing that the ECF campaign has exceeded the unconditional expected project value. When the crowd is very enthusiastic about the project/team ($\rho = 1.25$, continuous lines), the campaign goal is reasonable (x/x_0 around 1), the crowd is wise (σ small), and the success of the financing campaign has very little effect on the expected future value of

the project, which implies that the ratio (8) is close to one. For projects that are a priori unlikely ($\rho = 0.75$, dashed lines), it is surprising that the ECF campaign is successful and such a success modifies the perception of the future value possible outcomes. For that reason, the ratio (8) is large. When there is no wisdom in the crowd (black lines), the ECF campaign is expected to be successful when the campaign goal is low ($x < x_0$). Therefore, the ratio (8) is slightly larger than 1. For a larger value of x/x_0, greater than one, luck still plays a part in the success of a campaign that makes the progression of the ratio (8) slower than for the other cases.

This last result suggests that for projects that are a priori unlikely to succeed, a successful ECF campaign is a very good way of adding value and placing the startup in a much stronger position in future financing rounds.
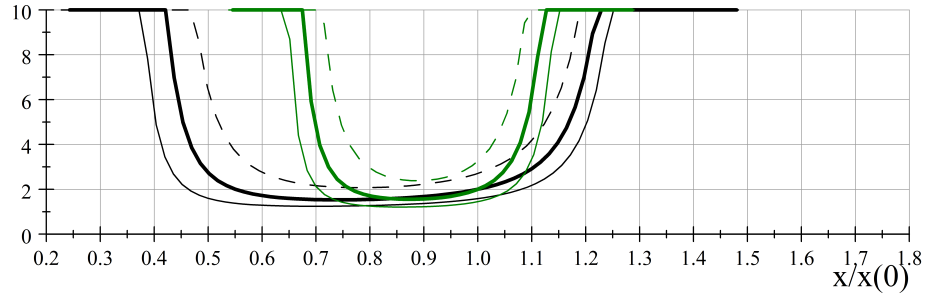

Figure 5: Effect of a successful ECF campaign on the expected project future value.   This figure shows by how much the project's expected future value increases if the ECF campaign is successful. The parameters are $x_0 = 1$, $\sigma \in \{ \frac{3}{4}$ (black), $\frac{1}{2}$ (green)$\}$; $\rho \in \{0.75$ (dashed), 1 (thick), 1.25 (continuous)$\}$ and $\varepsilon = 30\%$. For this specific illustration, we assume that the total funding X_{T_1} is normally distributed with an expectation (4) and a standard deviation (5). For clarity, the curves are truncated when the ratio $\xi(x)$ exceed 10.

2.3.2   Startup failure probability

Startup failure occurs if the future project value $V_{T_2}^{\text{ECF}}$ at time T_2 falls below a critical threshold c. One way to parametrize the critical value is to express it in terms of deviation from the expected future project value, considering the volatility. More precisely, let the critical threshold be j standard deviations away from the expected future value:

$$c = \mu_V - j\sigma_V.$$

Theorem 2 *Assume that the future project value $V_{T_2}^{ECF}$ is a Gaussian random variable. The startup failure probability of an entrepreneur who initially chooses an ECF campaign as a first source of funding is*

$$\mathbb{P}\left(X_{T_1} < x \ or \ \{X_{T_1} \geq x \ and \ V_{T_2}^{ECF} < c\}\right) = F + (1-F)\,\Phi\left(-\frac{\mu_V}{\sigma_V}\frac{F}{1-F} - j\right) \tag{9}$$

where Φ is the cumulative distribution function of a standard normal random variable and F represents the ECF failure probability. Moreover, given that the ECF campaign is successful, the startup failure probability becomes

$$\mathbb{P}\left(V_{T_2}^{ECF} < c \,\big|\, X_{T_1} \geq x\right) = \Phi\left(-\frac{\mu_V}{\sigma_V}\frac{F}{1-F} - j\right). \tag{10}$$

The proof is in Appendix 4. The first term in Equation (9) is the failure probability of the ECF campaign. Since this probability is relatively high for many projects, the failure probability of an entrepreneur going through an ECF campaign is mainly driven by the ECF campaign failure, that is, the first term is much larger than the second one. On the other hand, Equation (10) presents the startup failure probability as conditional on the success of the ECF campaign. It is clear that this probability is smaller than the first one, and the difference is greater the larger F is.

Figure 6 presents the startup failure probability (9) and the conditional startup failure probability given that the ECF campaign is successful (10), with both expressed as a function of the ECF campaign failure

probability (6)

$$F = F_{Z_{T_1}} \left(\frac{x - m}{s} \right).$$

An interesting situation arises for projects with high likelihood of campaign failure: if the ECF campaign turns out to be successful, its likelihood of startup failure is drastically reduced. This suggests a motivation for a crowdfunding campaign similar to those launched through reward-based crowdfunding platforms such as Kickstarter: let the market verify the idea. Risky, but highly rewarding if it does.
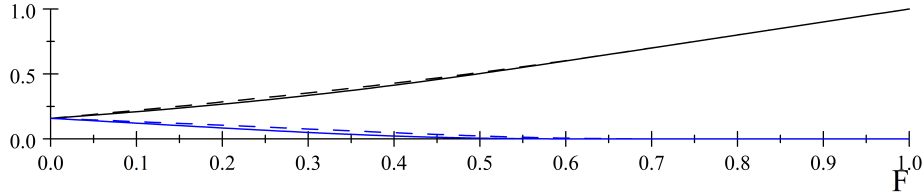

Figure 6: Startup failure probability and conditional probability seen as a function of the ECF failure probability. This figure presents the startup failure probability (9 – black lines) seen as a function of the ECF failure probability F. The critical threshold parameter $j = 1$, and the ratio of the expected future project value over its volatility $\frac{\mu_V}{\sigma_V}$ belongs to $\{1.5 \text{ (continuous)}, 1 \text{ (dashed)}\}$. The blue lines represent the startup failure probabilities (10 – blue lines), given that the ECF campaign is successful.

3   Seeking funds from an angel

In this section we consider the value added by seeking angel funding. The usual wisdom around angel investing (Ardichvili et al. 2002, Dutta & Folta 2016, Politis 2008) suggests that angels can add value, such as by reducing the chance of failure very early on, but also that they charge a lot of equity to compensate for that. We model this as follows:

Axiom 2 *Having an angel (or a group of angels acting together) financing an early-stage project can significantly increase its profitability.*

   1. the expected future project value is increased by a factor of $a \geq 1$:

$$\mu_V^{Angel} = \mathrm{E}_0^{\mathbb{P}} \left[V_{T_2}^{Angel} \right] = a\mu_V.$$

   2. The angel's share of the company is α_{Angel} which is typically larger than the crowd's share α_{ECF}.

3.1   Start-up failure probability

Theorem 3 *The startup failure probability for an entrepreneur who initially chooses an angel as the first source of funding is*

$$\mathbb{P} \left(V_{T_2}^{Angel} < c \right) = \mathbb{P} \left(\frac{V_{T_2}^{Angel} - a\mu_V}{\sigma_V} < \frac{c - a\mu_V}{\sigma_V} \right) = \Phi \left(-(a-1)\frac{\mu_V}{\sigma_V} - j \right) \tag{11}$$

where Φ is the cumulative distribution function of a standard normal random variable.

Of course, without the risk of an ECF campaign failure, the startup failure probability failure $\mathbb{P} \left(V_{T_2}^{\mathrm{Angel}} < c \right)$ Equation (11) is smaller than $\mathbb{P} \left(V_{T_2}^{\mathrm{ECF}} < c \right)$ (9). However, there is no clear dominance when (11) is compared to the conditional startup failure probability $\mathbb{P} \left(V_{T_2}^{\mathrm{ECF}} < c \middle| X_{T_1} \geq x \right)$ given that the ECF campaign is successful (10).

Corollary 1 *If the entrepreneur estimates the ECF failure probability F to be below $1 - \frac{1}{a}$, then her or his best option is to seek funding from an angel. Technically speaking,*

$$\mathbb{P} \left(V_{T_2}^{ECF} < c \middle| X_{T_1} \geq x \right) > \mathbb{P} \left(V_{T_2}^{Angel} < c \right) \text{ if and only if } F < 1 - \frac{1}{a}. \tag{12}$$

The proof is provided in Appendix 4.

Figure 7 presents an interesting case in which the startup failure probability of a successful ECF campaign (10) is compared with the startup failure probability of a project financed by an angel (11). If the angel does not bring value ($a = 1$, red curves), the latter is more important than the former. If the angel does bring value to the project ($a > 1$, green curves), the entrepreneur should still chose the ECF route, if and only if the ECF failure probability is larger than

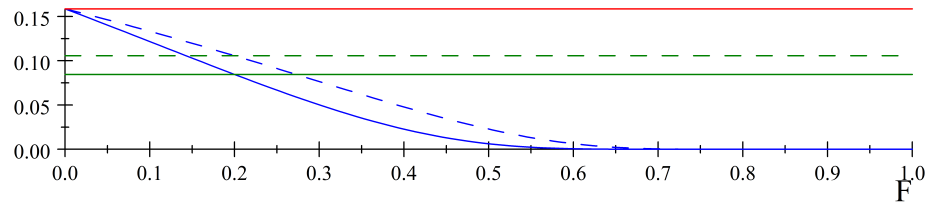$$F^*(a) = 1 - \frac{1}{a}. \tag{13}$$

Figure 7: Startup failure probabilities, seen as a function of the ECF failure probability. This figure presents the startup failure probabilities (10 - blue lines), given that the ECF campaign is successful. The red ($a = 1$) and green ($a = 1.25$) lines are the startup failure probabilities (9) for entrepreneurs seeking funding through an angel. The critical threshold parameter $j = 1$ and the ratio of the expected future project value over its volatility $\frac{\mu_V}{\sigma_V}$ belongs to $\{1.5 \text{ (continuous)}, 1 \text{ (dashed)}\}$.

Figure 8 shows the ECF failure probability threshold as a function of the factor a. For a given pair (a, F), the entrepreneur should seek financing through an angel if (a, F) is below the red curve. We note that even if the angel does bring value (a larger than 1), the entrepreneur is better off with the ECF for a relatively low ECF failure probability.
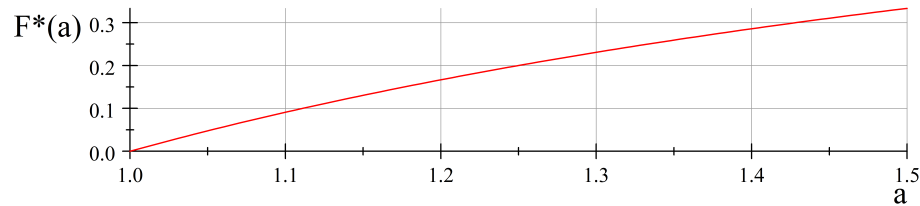

Figure 8: F*. This figure presents the ECF failure probability threshold (13) below which an entrepreneur who wants to optimize the startup failure probability should opt for financing via an angel. This threshold depends on the parameter a which characterizes the angel's contribution in increasing the value of the project and, consequently, in reducing the startup failure probability. Above this threshold, the entrepreneur must decide whether to accept the risks of an ECF campaign. If the ECF campaign is successful, then the startup failure probability will be smaller than with an angel.

3.2 The entrepreneur's expected share of the project

Theorem 4 *For an entrepreneur who chooses an ECF campaign, her or his expected share of the future project value is*

$$
(1 - \alpha_{\text{ECF}}) \, \mathrm{E}_0^{\mathbb{P}} \left[V_{T_2}^{ECF} \mathbb{I}_{V_{T_2}^{ECF} > c} \mathbb{I}_{X_{T_1} > x} \right]
$$
$$
= (1 - \alpha_{\text{ECF}}) \left(\sigma_V (1 - F) \, \varphi \left(\frac{F}{1 - F} \frac{\mu_V}{\sigma_V} + j \right) + \mu_V \Phi \left(\frac{F}{1 - F} \frac{\mu_V}{\sigma_V} + j \right) \right)
$$

where φ and Φ are the density and the cumulative distribution functions of a standard normal random variable respectively.

The proof is provided in Appendix 4.

Theorem 5 *In the case where the entrepreneur is financed through an angel, the entrepreneur's expected share of the future project value is*

$$(1 - \alpha_{\text{Angel}}) \, \mathrm{E}_0^{\mathbb{P}} \left[V_{T_2}^{\text{Angel}} \mathbb{I}_{V_{T_2}^{Angel} > c} \right]$$
$$= (1 - \alpha_{\text{Angel}}) \left(\sigma_V \varphi \left((a-1) \frac{\mu_V}{\sigma_V} + j \right) + a\mu_V \Phi \left((a-1) \frac{\mu_V}{\sigma_V} + j \right) \right).$$

A detailed proof is provided in Appendix 4.

Corollary 2 *If the entrepreneur wants to maximize her or his expected share of the future project value, then there is a critical ECF failure probability $F^{**}(a) < 1 - \frac{1}{a}$ below which she or he is better off seeking funds via an angel and above which she or he is better off chosing an ECF campaign.*

A detailed proof is provided in Appendix 4.

There is a lot of value in opting for an ECF campaign, and only in extreme cases is the entrepreneur clearly better off with angels. This is consistent with the observation (Wang et al. 2019) that angel investors increasingly use ECF platforms themselves to make their investments. This suggests that they too see the value in having the crowds verify the idea.

4   Conclusion

Early-stage finance is complex, chaotic and opaque: startups, investors and deals can dramatically differ from one another, and for the most part, deals are conducted in private. In recent years, governments around the world have prioritized innovation and entrepreneurship, encouraging investment in startups via generous tax incentive schemes. Equity Crowdfunding takes advantage of these new attitudes (and tax breaks) and seeks to level the field to provide access to early-stage companies by allowing the public to invest relatively small amounts in potentially many startups. These platforms have now become so popular that in some places like the UK, they already account for majority of early-stage deals.

Crowdfunding is also popular with researchers, as the data generated is much more detailed and precise than many other sectors of finance. Platforms know what each user is looking at, when she or he decides to invest or not, while gathering data on entrepreneurs, campaigns and investors. Previous studies have looked at what features of the startup and the campaign are associated with campaign success (Vulkan et al 2016); the rationality (or irrationality) of the crowds' investment decisions (Astebro et al 2018, Estrin & Khavul 2016) and how different types of entrepreneurs set campaign goals (Hellmann et al. 2019).

However, until now there has not been much in the way of theory providing a general framework with which to consider decisions made by entrepreneurs and large angel investors. The purpose of this paper is to try and close this gap.

Like Strausz (2017) and Ellman & Hurkens (2019), our model shows how the wisdom of the crowd has potentially massive positive impact on the value of startups, especially when there is little else to go by. The second part of our model, which shows how the value of the crowd compares with that of an angel investor is reminiscent of the founder's dilemma (Wasserman 2008) in that entrepreneurs choose between a larger share or more control, although unlike in Wasserman, in our model the value of the crowd is almost always larger, suggesting a clear difference between early-stage and scale-up financing. The interplay between angel investing and ECF predicted by our model seems to be consistent with the rough trends in the UK to the extent that these can be seen from the data so far in Beauhurst (2020).

Like any economic model, ours is restricted by our assumptions. We hope nevertheless that it can be useful for entrepreneurs thinking about where to raise money and how; for investors and policymakers, in understanding what the real value of the crowd is and when it is more important. And finally, it is also useful for ECF platforms that are still developing their business models and considering their relationship

with angel investors and how it might impact the "real crowd." Our work shows that this co-investing could be positive, especially for unknown and unusual startups.

At the time of writing, five firms are worth over a trillion dollar each. All these companies were launched in the last 30 years. Entrepreneurship continues to shape our world faster than ever before. Understanding the incentives of entrepreneurs and investors at early stages of the startup's existence is therefore of great importance. We hope this paper is a step in that direction.

Appendix

Proofs

Proof of Theorem 1. The expected project value at time T_2, $\mu_V = \mathrm{E}^{\mathbb{P}}\left[V_{T_2}^{\mathrm{ECF}}\right]$, may be broken down according to the success or the failure of the campaign:

$$
\begin{aligned}
\mu_V =& \mathrm{E}^{\mathbb{P}}\left[V_{T_2}^{\mathrm{ECF}}\mathbb{I}_{X_{T_1}\geq x} + V_{T_2}^{\mathrm{ECF}}\mathbb{I}_{X_{T_1}<x}\right]\\
=& \mathrm{E}^{\mathbb{P}}\left[V_{T_2}^{\mathrm{ECF}}\,|X_{T_1}\geq x\right]\mathbb{P}\left[X_{T_1}\geq x\right] + \underbrace{\mathrm{E}^{\mathbb{P}}\left[V_{T_2}^{\mathrm{ECF}}\,|X_{T_1}<x\right]}_{=0}\mathbb{P}\left[X_{T_1}<x\right].
\end{aligned}
$$

$\square$

Proof of Theorem 2. At time $t = 0$, if the entrepreneur launches an ECF campaign, the startup failure probability is the probability of an ECF campaign failure, or the ECF campaign is a success but the future project value is below the threshold:

$$
\mathbb{P}\left(\{X_{T_1}<x\}\text{ or }\{X_{T_1}\geq x\text{ and }V_{T_2}^{\mathrm{ECF}}<c\}\right) = \mathbb{P}\left(X_{T_1}<x\right) + \mathbb{P}\left(V_{T_2}^{\mathrm{ECF}}<c\big|\,X_{T_1}\geq x\right)\mathbb{P}\left(X_{T_1}\geq x\right).
$$

Moreover,

$$
\begin{aligned}
\mathbb{P}\left(V_{T_2}^{\mathrm{ECF}}<c\big|\,X_{T_1}\geq x\right) =& \mathbb{P}\left(\left.\frac{V_{T_2}^{\mathrm{ECF}}-\frac{\mu_V}{\mathbb{P}\left(X_{T_1}\geq x\right)}}{\sigma_V} < \frac{\mu_V - j\sigma_V - \frac{\mu_V}{\mathbb{P}\left(X_{T_1}\geq x\right)}}{\sigma_V}\right|X_{T_1}\geq x\right)\\
=& \Phi\left(\frac{\mu_V}{\sigma_V}\left(1-\frac{1}{\mathbb{P}\left(X_{T_1}\geq x\right)}\right)-j\right).
\end{aligned}
$$

$\square$

Proof of Corollary 1.

$$
\begin{aligned}
& \mathbb{P}\left(V_{T_2}^{\mathrm{ECF}}<c\big|\,X_{T_1}\geq x\right) > \mathbb{P}\left(V_{T_2}^{\mathrm{Angel}}<c\right)\\
\iff & \Phi\left(-\frac{\mu_V}{\sigma_V}\frac{F}{1-F}-j\right) > \Phi\left(-(a-1)\frac{\mu_V}{\sigma_V}-j\right)\\
\iff & -\frac{\mu_V}{\sigma_V}\frac{F}{1-F}-j > -(a-1)\frac{\mu_V}{\sigma_V}-j\text{ since }\Phi\text{ is strictly increasing,}\\
\iff & F < \frac{a-1}{a}.
\end{aligned}
$$

$\square$

Proof of Theorem 4. First, note that

$$
\int_x^\infty z\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)dz = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right) = \varphi(x) \tag{14}
$$

where φ is the density function of a standard normal random variable.

$$
\mathrm{E}_0^{\mathbb{P}}\left[V_{T_2}^{\mathrm{ECF}}\mathbb{I}_{V_{T_2}^{\mathrm{ECF}}>c}\mathbb{I}_{X_{T_1}\geq x}\right]=\mathrm{E}_0^{\mathbb{P}}\left[V_{T_2}^{\mathrm{ECF}}\mathbb{I}_{V_{T_2}^{\mathrm{ECF}}>c}\bigg|\,X_{T_1}\geq x\right]\underbrace{\mathbb{P}\left[X_{T_1}\geq x\right]}_{=1-F}
$$

$$
=\sigma_V\mathrm{E}_0^{\mathbb{P}}\left[\frac{V_{T_2}^{\mathrm{ECF}}-\frac{\mu_V}{1-F}}{\sigma_V}\mathbb{I}_{\frac{V_{T_2}^{\mathrm{ECF}}-\frac{\mu_V}{1-F}}{\sigma_V}>\frac{c-\frac{\mu_V}{1-F}}{\sigma_V}}\bigg|\,X_{T_1}\geq x\right](1-F)
$$

$$
+\frac{\mu_V}{1-F}\mathrm{E}_0^{\mathbb{P}}\left[\mathbb{I}_{\frac{V_{T_2}^{\mathrm{ECF}}-\frac{\mu_V}{1-F}}{\sigma_V}>\frac{c-\frac{\mu_V}{1-F}}{\sigma_V}}\bigg|\,X_{T_1}\geq x\right](1-F)
$$

$$
=\sigma_V\left(1-F\right)\varphi\left(\frac{c-\frac{\mu_V}{1-F}}{\sigma_V}\right)+\mu_V\Phi\left(-\frac{c-\frac{\mu_V}{1-F}}{\sigma_V}\right)
$$

$$
=\sigma_V\left(1-F\right)\varphi\left(\frac{F}{1-F}\frac{\mu_V}{\sigma_V}+j\right)+\mu_V\Phi\left(\frac{F}{1-F}\frac{\mu_V}{\sigma_V}+j\right)\ \text{ since }\varphi\left(-x\right)=\varphi\left(x\right)
$$

$$
=\sigma_V\left(1-F\right)\varphi\left(\left(\frac{1}{1-F}-1\right)\frac{\mu_V}{\sigma_V}+j\right)+\mu_V\Phi\left(\left(\frac{1}{1-F}-1\right)\frac{\mu_V}{\sigma_V}+j\right).
$$

$\square$

Proof of Theorem 5. From Equation (**??**) and because $\varphi\left(-x\right)=\varphi\left(x\right)$,

$$
\mathrm{E}_0^{\mathbb{P}}\left[V_{T_2}^{\mathrm{Angel}}\mathbb{I}_{V_{T_2}^{\mathrm{Angel}}>c}\right]=\sigma_V\mathrm{E}_0^{\mathbb{P}}\left[\frac{V_{T_2}^{\mathrm{Angel}}-a\mu_V}{\sigma_V}\mathbb{I}_{\frac{V_{T_2}^{\mathrm{Angel}}-a\mu_V}{\sigma_V}>\frac{c-a\mu_V}{\sigma_V}}\right]+a\mu_V\left(1-\Phi\left(\frac{c-a\mu_V}{\sigma_V}\right)\right)
$$

$$
=\sigma_V\varphi\left(\frac{c-a\mu_V}{\sigma_V}\right)+a\mu_V\Phi\left(-\frac{c-a\mu_V}{\sigma_V}\right)
$$

$$
=\sigma_V\varphi\left((a-1)\frac{\mu_V}{\sigma_V}+j\right)+a\mu_V\Phi\left((a-1)\frac{\mu_V}{\sigma_V}+j\right).
$$

$\square$

Proof of Corollary 2. We now compare the entrepreneur's expected share depending on her or his funding source, that is (14) versus (14). Because the critical threshold $c=\mu_V-j\sigma_V>0$ is positive, $\frac{\mu_V}{\sigma_V}-j>0$.

Note that

$$
(1-\alpha_{\mathrm{Angel}})\,\mathrm{E}_0^{\mathbb{P}}\left[V_{T_2}^{\mathrm{Angel}}\mathbb{I}_{V_{T_2}^{\mathrm{Angel}}>c}\right]<(1-\alpha_{\mathrm{ECF}})\,\mathrm{E}_0^{\mathbb{P}}\left[V_{T_2}^{\mathrm{ECF}}\mathbb{I}_{V_{T_2}^{\mathrm{ECF}}>c}\right]
$$

if and only if

$$
\frac{1-\alpha_{\mathrm{Angel}}}{1-\alpha_{\mathrm{ECF}}}\frac{1}{1-F}<\frac{\varphi\left(\left(\frac{1}{1-F}-1\right)\frac{\mu_V}{\sigma_V}+j\right)+\frac{1}{1-F}\frac{\mu_V}{\sigma_V}\Phi\left(\left(\frac{1}{1-F}-1\right)\frac{\mu_V}{\sigma_V}+j\right)}{\varphi\left((a-1)\frac{\mu_V}{\sigma_V}+j\right)+a\frac{\mu_V}{\sigma_V}\Phi\left((a-1)\frac{\mu_V}{\sigma_V}+j\right)}. \tag{15}
$$

Note that $g\left(y\right)=\varphi\left((y-1)\frac{\mu_V}{\sigma_V}+j\right)+y\frac{\mu_V}{\sigma_V}\Phi\left((y-1)\frac{\mu_V}{\sigma_V}+j\right)$ is an increasing function of y. Indeed, since $\varphi'\left(x\right)=-x\varphi\left(x\right)$, then

$$
g'\left(y\right)=\frac{\partial}{\partial y}\left\{\varphi\left((y-1)\frac{\mu_V}{\sigma_V}+j\right)+y\frac{\mu_V}{\sigma_V}\Phi\left((y-1)\frac{\mu_V}{\sigma_V}+j\right)\right\}
$$

$$
=\frac{\mu_V}{\sigma_V}\varphi'\left((y-1)\frac{\mu_V}{\sigma_V}+j\right)+\frac{\mu_V}{\sigma_V}\Phi\left((y-1)\frac{\mu_V}{\sigma_V}+j\right)+y\left(\frac{\mu_V}{\sigma_V}\right)^2\varphi\left((y-1)\frac{\mu_V}{\sigma_V}+j\right)
$$

$$
=\frac{\mu_V}{\sigma_V}\left\{\Phi\left((y-1)\frac{\mu_V}{\sigma_V}+j\right)+\left(\frac{\mu_V}{\sigma_V}-j\right)\varphi\left((y-1)\frac{\mu_V}{\sigma_V}+j\right)\right\}>0
$$

because Φ is a cumulative distribution function and φ is a density function, both of them are always positive. Moreover, the right-hand side of the inequality (15) is worth one if $F=1-\frac{1}{a}$. As the left-hand side is less than one whenever $\alpha_{\mathrm{Angel}}\geq\alpha_{\mathrm{ECF}}$, the threshold $F^{**}\left(a,\frac{1-\alpha_{\mathrm{Angel}}}{1-\alpha_{\mathrm{ECF}}}\right)$ below which the entrepreneur who wants to maximise his share of the project value should choose the angel is below $1-\frac{1}{a}$. $\square$

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