

A dynamic stochastic approach for open-pit mine planning

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G-2018-87

October 2018

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Citation suggérée: A. Rimélé, R. Dimitrakopoulos, M. Gamache (Octobre 2018). A dynamic stochastic approach for open-pit mine planning, Rapport technique, Les Cahiers du GERAD G-2018-87, GERAD, HEC Montréal, Canada.

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Suggested citation: A. Rimélé, R. Dimitrakopoulos, M. Gamache (October 2018). A dynamic stochastic approach for open-pit mine planning, Technical report, Les Cahiers du GERAD G-2018-87, GERAD, HEC Montréal, Canada.

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A dynamic stochastic approach for open-pit mine planning

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October 2018

Les Cahiers du GERAD

G–2018–87

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Abstract: Over the last decade, geological uncertainty, its effects on long-term mine planning and methods for related risk management have been studied. However, the combined effect of geological and commodity price uncertainty has received less attention in the technical literature. A research experiment addressing both these sources of uncertainty is presented here, while accounting for their differences. In particular, while the current commodity price is known at the beginning of every new mining period, the geology, including mineral grades, metal content, material types and so on, remain uncertain, even when additional information becomes available. The proposed method first uses a two-stage model to manage the geological uncertainty leading to a scenario-independent extraction sequence and, based on different metal production targets, a pool of subsets of mining blocks is also precomputed for every period. Then, a stochastic dynamic algorithm is developed and employed to define the best policy in terms of metal production targets to follow, depending on the evolution of the related commodity price. This policy follows the scenario tree of the commodity price, as it is scenario-dependent (price only) with non-anticipativity constraints, similar to an operator adapting to a fluctuating market. This new approach is tested on a small case study and reveals the counter-intuitive combined effects of both sources of uncertainty.

Keywords: Open pit mine planning, geological and market uncertainty, stochastic model, stochastic dynamic programming

1 Introduction

A realistic long-term mine production plan is required to define a sequence of extraction of materials from a mineral deposit and forecast metal production while maximising net present value over the life of the related mine. Due to the high infrastructure costs and investments, lower revenues than expected or delayed returns on investment can compromise expectations, plans and mining projects. Over the past decades, several optimization approaches and mixed integer programming (MIP) models have been developed (Hustrulid et al 2013; Osanloo et al 2008; Newman et al. 2010; Dimitrakopoulos 2018). In these models, the ore body is discretized into mining blocks, and the objective of the optimization is to schedule the extraction of these blocks per year of production to maximize the net present value over the life-of-mine. The conventional methods are deterministic (Hustrulid et al 2013; Dimitrakopoulos 2018) as they do not account for uncertainty, but they are still widely used in the industry and distributed by commercial schedulers (e.g., GEOVIA Whittle Strategic Mine Planning). Nonetheless, uncertainties play a significant role in long-term mine planning and ignoring them while planning and forecasting production and cashflows has been shown to lead to suboptimal results. The first main source of uncertainty relates to the description of pertinent geological attributes of the deposits being mined, such as grades, material types, geometallurgical properties, and so on, for the blocks representing the deposit. Conventional estimation methods provide average values for related rock properties that tend to smooth grades and do not represent the local variability of the materials mined or provide uncertainty quantification needed for mine planning optimization (e.g., Godoy 2003; Dimitrakopoulos et al. 2002). For this reason, stochastic simulations (Goovaerts 1997) have been used to replace the representation of mineral deposits considered, a group of which is used to quantify geological uncertainty and serve as the input to stochastic integer programming (SIP) models and have proven effective in managing geological uncertainty as well as in quantifying and referring risk in production forecasts (Ramazan and Dimitrakopoulos 2005, Godoy and Dimitrakopoulos 2004, Dimitrakopoulos 2011, Ramazan and Dimitrakopoulos 2013). The information provided by the simulations about the grade variability allows risk to be managed and the net present value (NPV) to be maximized. Compared to a deterministic approach, the stochastic framework consistently increases the NPV (up to 25%: Dimitrakopoulos 2011, Ramazan and Dimitrakopoulos 2013, Spleit and Dimitrakopoulos 2018, Navarra et al. 2016, Montiel et al. 2016; Spleit and Dimitrakopoulos 2018, Vallejo and Dimitrakopoulos 2018, others) as well as manages the related technical risk in the presence of geological uncertainty. While the advantages of such SIPs are numerous, their solving complexity is substantial, particularly when industrial mining complexes that include several mines and processing streams that are optimized simultaneously. Recent developments have sought to improve this broader framework and optimization models thus render them applicable to complex case studies (Lamghari and Dimitrakopoulos 2012, Gilani and Sattarvand 2016, Lamghari and Dimitrakopoulos 2016, Montiel and Dimitrakopoulos 2015, Goodfellow and Dimitrakopoulos 2016, 2017). These studies have proven the viability and advantages of stochastic models in managing the geological uncertainty in several real-life cases.

While geological uncertainty and its consequences for long-term mine planning have been understood and addressed, the influence of commodity price uncertainty remains to be further explored and fully integrated into the strategic mine planning optimization process, jointly with geological uncertainty. Sabour and Dimitrakopoulos (2007) consider both uncertainties with a sensitivity analysis method that allows them to rank pit designs that were generated from the commercial software Whittle. Meagher et al. (2009) propose a method that creates a more risk-resilient pushback design by assigning modified dollar values at the mining blocks that depend on price variability. Asad and Dimitrakopoulos (2013) implemented a parametric maximum flow algorithm to optimize pushbacks in a stochastic framework. Their method provides accurate results efficiently; however, it would be substantially more complicated to adapt to the actual mine production scheduling optimization. Del Castillo and Dimitrakopoulos (2014) integrate both uncertainties in order to optimize the ultimate pit limit based upon a probabilistic analysis over the life-of-mine, based on a set of combined scenarios. Kizilkale and Dimitrakopoulos (2014) present a dynamic programming approach that optimizes the mining rates under market uncertainty; while the approach and results provide interesting insights to production rates, the approach does not ensure that resulting production rates may be feasible given geological uncertainty.

The optimization method presented herein considers both geological and market uncertainties with a method that provides a life-of-mine production schedule that is scenario independent with regards to geological uncertainty and scenario dependant with respect to the non-anticipativity of the commodity price. The idea behind the latter is to provide the optimization model with the possibility of adapting to the unveiled commodity price, however, only with reasonable options in terms of changes in metal productions targets as well as feasible mine designs and production plans. The method employs a combination of a two-stage stochastic integer program and a stochastic dynamic programming algorithm. In the following sections, the proposed optimization method is presented in detail. Then, an application of the method to a case study explores the practical aspects and related concepts. Conclusions and recommendations follow.

2 Method

To develop a method that is resilient to both geological and price uncertainties for long-term open-pit mine planning, a key consideration is the intrinsic difference of these two sources of uncertainty. While new information is revealed during production and operators can adapt accordingly, it is important to note that (a) when considering the related commodity price, it is known at a given point in time; however, (b) the geology and the pertinent rock properties affecting production planning and scheduling remain uncertain, despite the collection of additional information. The latter unavoidable uncertainty requires that a uniquely defined, scenario-independent, mine production plan and schedule are generated and can be followed, and this is accommodated through the use of a two-stage stochastic integer programming model with fixed recourses (Birge and Louveaux 2011). On the other hand, price uncertainty calls for more flexibility. At the beginning of a new period (year) the commodity price is considered known for the year and moving forward, production planning will differ if the commodity price is higher or lower than the previous year. As a result, assessing and quantifying the viability of a production plan given the price volatility through a two-stage SIP and price-independent decisions would be over-pessimistic, assuming that decisions are identical for different price evolutions. As a result, the modeling approach requires a stochastic multistage model with decision variables that are geology-independent but price-dependent (branching). Non-anticipativity constraints can prevent anticipating the yet-unrevealed price, and the model could provide different schedules for different price evolutions (a path in the scenario tree). A known limitation of such multistage models comes from the curse of dimensionality. In particular, the number of variables and constraints (exponential with the number of periods) induced by the scenario tree creates an extremely large model that becomes less solvable (Figure 1).

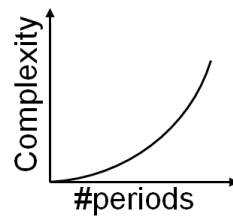


Figure 1: Complexity of the discussed stochastic multistage model.

In order to overcome this limit, the proposed approach is a stochastic dynamic algorithm (Bertsekas, 1995). The idea behind this algorithm is the following. Firstly, subsets of blocks corresponding to given metal targets are pre-computed with a typical two-stage SIP that considers the geological uncertainty (Section 2.1); then, a stochastic dynamic algorithm defines the best policy to follow (which subset of blocks to extract in the coming year) depending on the evolution of the price. The procedure involves computing a large number of SIPs to define those subsets of blocks (again, exponential in the number of periods); however, those SIPs are much simpler than the multistage model and become exponentially easier to solve, that is fewer periods and blocks remain in the models (Figure 2). The main drawback comes from the limited number of subsets of blocks picked by the dynamic algorithm, which leads to the feasible space becoming drastically restricted. Yet, operationally, this can be seen as an advantage, since it limits the possible extractions and results in fewer future designs to consider, which also addresses practical mine planning limitations.

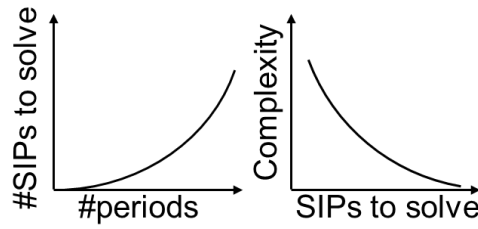


Figure 2: Stochastic dynamic approach: number of individual SIPs to solve (left) and complexity of the SIPs (right).

The next section explains in detail the pre-computing of the subsets of blocks used by the dynamic algorithm.

2.1 Preprocessing – pool of yearly subsets of blocks

As mentioned above, in order to simplify the proposed model and to limit the creation of different mine designs, a pool of possible subsets of blocks are pre-computed. These block subsets correspond to a general metal production target that is used to facilitate the optimization process and to construct a tree of possible extraction sequences as shown in Figure 3. The root node shown in the figure corresponds to the initial situation in which nothing has been extracted yet. This root node has n arcs oriented toward children nodes (two children in Figure 3). The children nodes correspond to variations in metal targets from -10% to $+10\%$ relative to an initial value. A SIP model, discussed below, is used to define the subset of blocks to be extracted in the current period at those nodes. Again, these nodes have children that have variations of metal production targets and are optimized to identify new subsets of blocks. Note that the subset of blocks from the parent have previously been removed, which makes the optimization model simpler to solve.

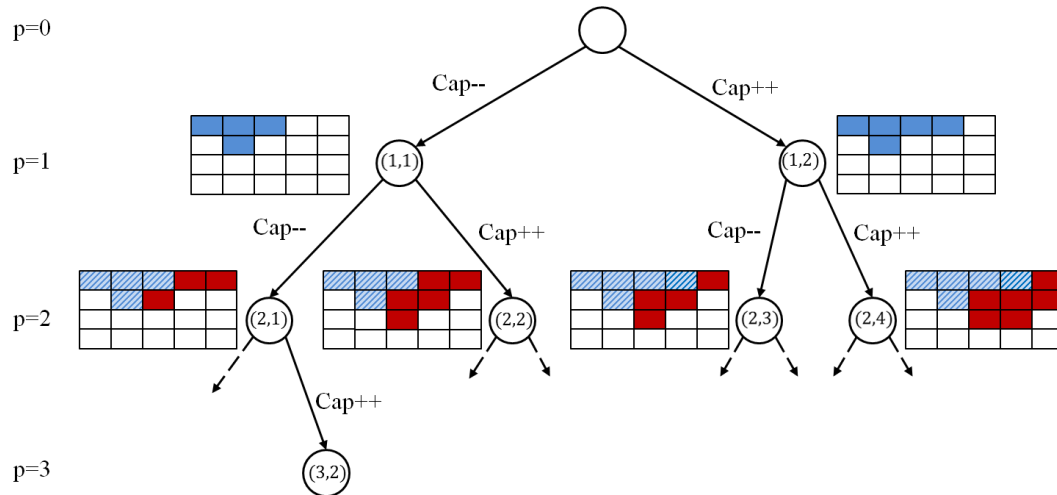


Figure 3: Computation of the pool of subsets of blocks, tree of metal production target Γ .

For a given period t and at a given node i , the subset of blocks to be extracted is obtained by solving the two-stage Open-pit Mine Planning SIP (OMP-SIP) formulation, integrating geological uncertainty, shown below. The first-stage binary decision variables $x_{b,p}$ denote if block $b \in \mathcal{B}$ has been extracted by period $p \in \mathcal{P}$ ($=1$) or not ($=0$), and second-stage continuous recourse variables $dev_{c,p,s}^{\pm}$ denote the deviations from the production targets of attribute $c \in \mathcal{C}$ at period p associated with the geological scenario $s \in \mathcal{S}$. Several parameters are used: $v_{b,p,s}$ denotes the value of block b , scenario s if extracted in period p ; $cost_{c,p}^{\pm}$ is the penalty cost associated to over (+) or under (−) production compared to the production target of attribute c in period p ($target_{c,p}^{\pm}$); $q_{c,b,s}$ denotes the quantity of attribute c of block b , geological s .

The objective function is

$$\max z = \frac{1}{|\mathcal{S}|} * \sum_{b \in \mathcal{B}} \sum_{p \in \mathcal{P}} \sum_{s \in \mathcal{S}} v_{b,p,s} (x_{b,p} - x_{b,p-1}) - \frac{1}{|\mathcal{S}|} * \sum_{p \in \mathcal{P}} \sum_{s \in \mathcal{S}} (cost_{c,p}^- * dev_{c,p,s}^- + cost_{c,p}^+ * dev_{c,p,s}^+) \quad (1)$$

Subject to

$$x_{b,p} - x_{b,p-1} \geq 0 \quad \forall b \in \mathcal{B}, \forall p \in \mathcal{P} \quad (2)$$

$$x_{b,p} \leq x_{b',p} \quad \forall b \in \mathcal{B}, \forall b' \in \mathcal{B}_b^-, \forall p \in \mathcal{P} \quad (3)$$

$$\sum_{b \in \mathcal{B}} \left(q_{c,b,s} * (x_{b,p} - x_{b,p-1}) \right) \mp dev_{c,p,s}^\pm \leq \text{target}_{c,p}^\pm \quad \forall c \in \mathcal{C}, \forall p \in \mathcal{P}, \forall s \in \mathcal{S} \quad (4)$$

$$x_{b,p} = x_{b,p}^* \quad \forall b \in \mathcal{B}, \forall p \in [1, t-1],$$

where $x_{b,p}^*$ is the solution obtained at node $\Gamma^{-1}((t, i))$

(5)

The objective function (1) maximizes the discounted cash flow while penalizing deviations from the production targets. For more details about the formulation, the reader is referred to Rim  l   et al. (2018). The reserve constraints (2) state that a given block can be extracted only once. The precedence or slope constraints (3) define that a block b can be extracted only if its set of direct predecessor blocks \mathcal{B}_b^- have previously been extracted. (4) are the mine production target constraints. Constraints (5) fix blocks that have previously been extracted in the tree of metal production target Γ .

The use of this model allows the subsets of blocks to be resilient to the geological uncertainty by managing the risk of violating production constraints and by guarantying the respect of feasibility constraints (such as mining reserve and slope constraints). Note that at every node in the metal production target tree, the subset of blocks to be extracted within the period is not greedily optimized because all of the remaining periods are included in the OMP-SIP(t, i) model. Even if, at a given node, one is only interested in determining the blocks to extract during the corresponding period, having a long-term vision is essential for maximizing the life-of-mine discounted cash flow and for satisfying the constraints.

Now that a pool of block subsets is available, a stochastic dynamic algorithm can use it to define the best extraction policy to follow with regards to the unveiling of the commodity price.

2.2 Model – stochastic dynamic algorithm

This section presents the proposed stochastic dynamic algorithm, also referred to as the probabilistic sequential decision process. Its objective is to provide a guide (a policy) to follow depending on the evolution of the commodity price. More than a policy, which is likely to change with the operations, it provides a less-pessimistic evaluation of a life-of-mine plan, since it lets the policy change for different prices. Similar to any dynamic algorithm, a stochastic dynamic algorithm is defined by a horizon and a time step, state variables, decisions to be taken, a cost function, a transition function and Bellman equations (recursive equations). Here, the horizon is the life-of-mine, and the time step a year of extraction.

State variables

$$X_p = \left\{ (B_{p-1,i}, cp_p) \right\} \cup \Delta$$

The state variables (current state of the system) is composed of the blocks already extracted $B_{p-1,i}$ and the revealed current commodity price cp_p or if the mine has already closed Δ .

Decisions

At every time step and for every state variable, the decision to be taken consists in choosing a subset of blocks $B_{p,j} \setminus B_{p-1,i}$ to extract $\forall (p, j); \left((p-1, i), (p, j) \right) \in A$ or $j = 0$, A being the set of arcs of the metal production target tree.

Transition function

The transition function is simply the update of the extracted blocks $B_{p,j}$ (directly from the decision) and the revealed commodity price: $X_{p+1} = (B_{p,j}, cp_{p+1}(\omega))$, where ω defines a realization of the random variable associated with the commodity price.

Cost function

The cost function is the discounted cash flow obtained by extracting the chosen subset of blocks under the revealed commodity price: $g_p(B_{p,j} \setminus B_{p-1,i}, cp_p)$.

Bellman equations

The optimal life-of-mine expected discounted cash flow (objective function) is denoted by $J_{\pi^*,p=1}(X_1 = (\emptyset, cp_1))$. Defined recursively, this objective function must respect the following conditions:

1. If the mine is already closed, it does not generate any additional cash flow:

$$J_{\pi^*,p}(\Delta) = 0 \quad \forall p \leq P$$

2. After the end of life-of-mine, no more cash flow is generated:

$$J_{\pi^*,P+1}(X_{P+1}) = 0$$

3. The maximized profit at a given period aims to maximize the immediate cash flow $g_p(B_{p,j} \setminus B_{p-1,i}, cp_p)$ and the expected profit of the following period (expected value regarding the random variable associated with the commodity price)

$$J_{\pi^*,p}(X_p = (B_{p-1,i}, cp_p)) = \max_{j: \left((p-1,i), (p,j) \right) \in A \text{ or } j=0} \left\{ g_p(B_{p,j} \setminus B_{p-1,i}, cp_p) + \mathbb{E}_\omega \left[J_{\pi^*,p+1}(X_{p+1} = (B_{p,j}, cp_{p+1}(\omega))) \right] \right\} \\ \forall X_p \neq \Delta, \forall p \leq P$$

The method used to solve this dynamic algorithm is the well-known back-propagation method, which starts at the last period and defines the optimal policy for every possible state. Thereafter, it back-propagates, which signifies that it considers the second to last period and, again, chooses the best policy. This can be done because the future profits for any chosen decision is known from the previous iteration. The process continues until the optimal policy has been fully defined (the back-propagation arrives at the first period).

3 Case study

3.1 Presentation

The basic aspects of the proposed approach are tested on a small case study. This case study consists of a small part of a copper orebody composed of 4,273 mining blocks, which must be scheduled in a maximum of 4 periods and sent either to the mill or the waste dump.

The geological uncertainty is represented by a set of 20 stochastically simulations realizations of the deposit.

The initial metal production target was set to 20Ktons per year. From one year to another, this target can either remain the same, decrease by 1Kton or increase by 1Kton and this is used in the metal production target to define the pool of subsets of blocks. This reasonably small variations (+ or -5%) is considered

to provide realistic variations in terms of metal production for this small size deposit; in general, it is not realistic to let the optimizer change the production significantly from one year to another (c.f. equipment forward sales, workforce, equipment, etc.). The SIPs used in the metal production target tree have all been solved with the commercial solver CPLEX v12.6.

The price uncertainty is modeled as a Markovian process with equal probabilities (see Figure 4) (Del Castillo and Dimitrakopoulos, 2014). At every time step, five equally probable variations of the copper price are considered: -30% , -15% , $+0\%$, $+15\%$ and $+30\%$. This simplistic model can be extended to include more complex price variations; however, it is sufficient for the purposes of this study and its focus on the proposed optimization framework.

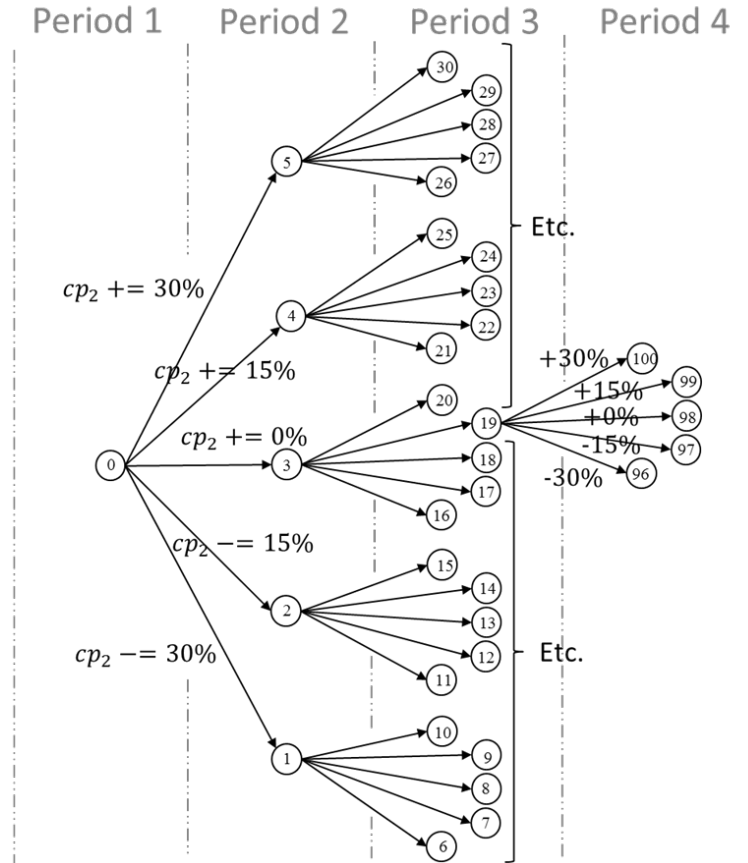


Figure 4: Commodity price evolution tree - Markovian process.

3.2 Results

The results obtained after back-propagation correspond to a policy of actions to take depending on the current commodity price and its history.

Figure 5 shows a part of the obtained policy. In the figure, the arcs labeled ++ have an increase of +1Kton of the metal target; the arcs labeled = mean that metal target remains constant, and the arcs labelled with a - indicate a decrease of -1Kton of metal target.

Figure 5, the upper part of the tree shows some dashed arcs. This indicates that the remainder of the branch has been cut off because the corresponding production plan was found to be unfeasible. Indeed, by consistently increasing the metal production targets, it becomes impossible for the mine to provide the required metal in the allotted time period, which results in violating the production constraints considered. This has the effect of decreasing the size of the decision tree, thus reducing computational complexity.

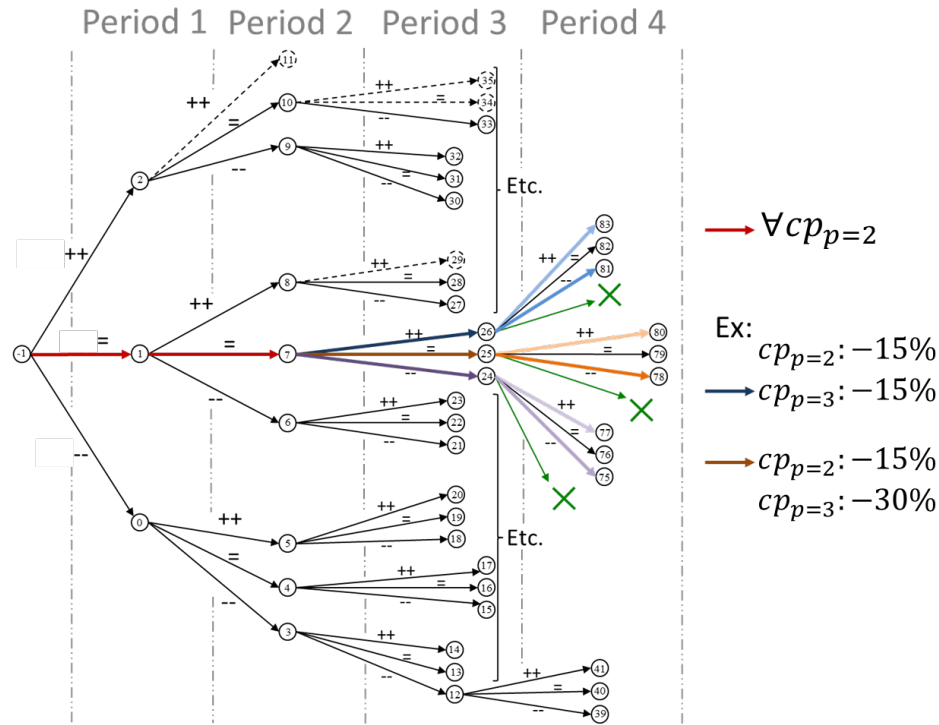


Figure 5: Optimal policy in terms of metal production target to follow depending on the commodity price.

Figure 6 shows a zoom-in around some price evolutions. In Period 1, the commodity price is already known (no price uncertainty), and the optimal policy keeps the metal production target at 20Ktons. For Period 2, surprisingly, it is always optimal to keep the metal production target constant for all the possible price evolutions. However, by Period 3, different policies are suggested depending on the price evolution. In some cases, where the price has constantly decreased (green cross), the most profitable option leads to end the life-of-mine at the end of Period 3.

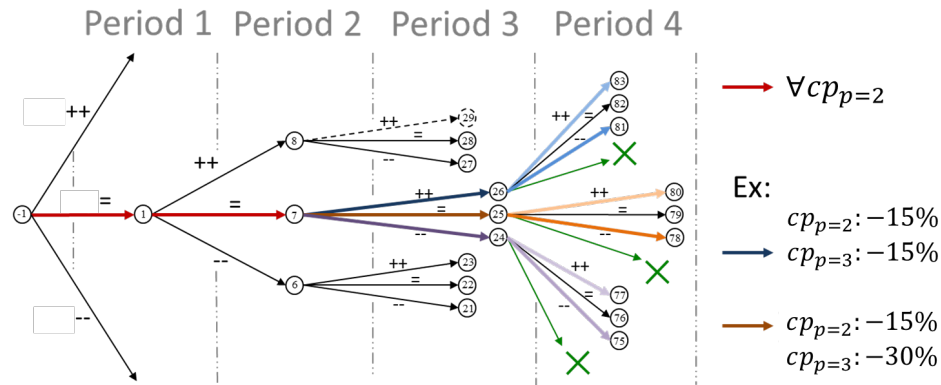


Figure 6: Optimal policy in terms of metal production target to follow, depending on the commodity price – zoom-in.

Figure 7 presents a clearer example of the obtained policy. Again, Graph (1) shows that, for all prices in Period 2 (blue arcs in “price tree”), it is best to keep the metal production target constant at 20Ktons (blue arc going to Node 7 in “metal target tree”). This can be surprising since, often, one would expect that an increase of price should lead to an increase of the metal production target to take advantage of the increased value of the extracted metal. Here, it appears that the fluctuations of price during the second period does not influence the optimal metal production target. This observation can be explained by different reasons.

One of these reasons may be the geological uncertainty that can influence the natural expectations with respect to the commodity price. For instance, trying to extract more metal given a higher price coupled with risky (highly uncertain and variable geological attributes) or low-grade accessible parts of the orebody may decrease the life-of-mine profits, because it may be overall better to wait for another year with potentially a lower commodity price to mine the related materials. Note that since an orebody is a finite resource, extracting more metal now means extracting less later, thus, even if the price increases at a given mining period, it may be wiser to keep the same production to be able to increase it later with, potentially an even better price. Graphs (2), (3) and (4) define explicitly the different price evolutions and corresponding policies in Period 3, if the price in Period 2 decreased by 30%. If, in Period 3, the price decreases again by 30% (Graph (2)), the optimal policy is to decrease the metal production by 5% (Node 24), result that corroborates the natural expectation. However, if the price only decreases by 15% (Graph (3)), then it is better to keep the metal production constant at 20Ktons (Node 25). Considering the severe situation of this scenario (−30% in Period 2 and −15% in Period 3), this result is less obvious. Another aspect, worth mentioning here, is the possibility to close the mine if the economic situation becomes too difficult. Because of that, even if the price becomes critical low, it may be better to keep extracting as much (or even increase the production). If the situation gets even worse the following year, the mine can be closed to prevent negative cash flow, else if the situation becomes better, the remaining metal can continue being extracted. Finally, if the price remains constant or increases (Graph (4)), it becomes optimal to increase the metal production target by 5% (Node 26). Having made explicit a portion of the global optimal policy helps understanding the complex dynamic that lead the best metal production target at every mining period. Not only, it is impacted by the variation of price but also other factors among which a finite resource, an uncertain supply and/or the option to prematurely stop the operations have non-trivial effects that can hardly be guessed by considering them independently.

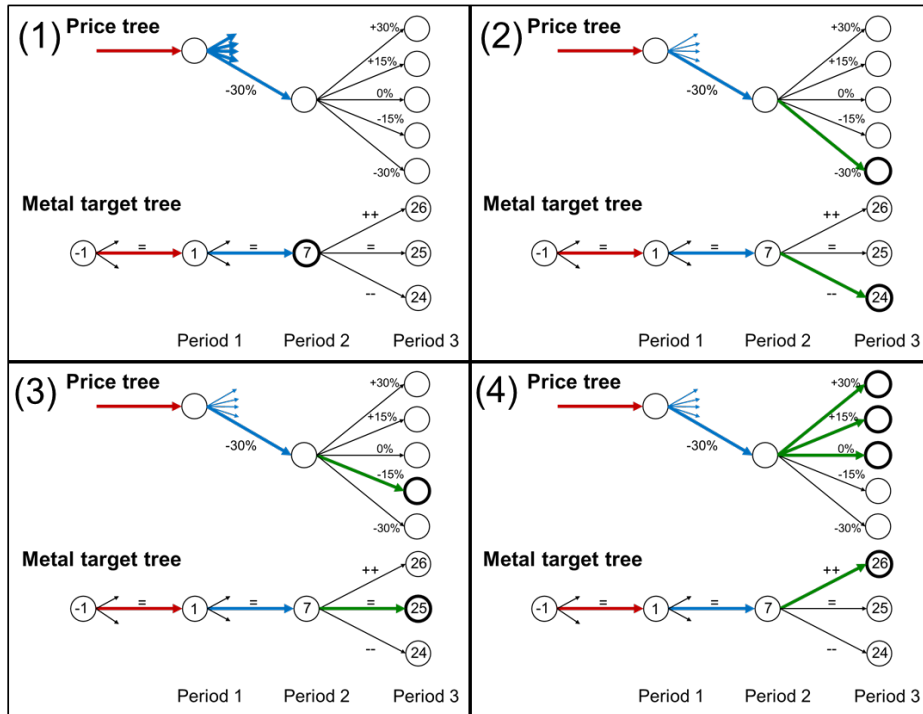


Figure 7: Details about a part of the obtained policy.

Figure 8 shows the different designs (cross sections) of the extraction sequences of the three policies previously detailed at Period 3 (nodes 24, 25, 26 in the metal target tree in Figure 7). Since the only difference of policy occurred in Period 3, the mining blocks extracted in Periods 1 and 2 are identical. One may note some of the changes in Period 3, circled in red in the cross-sections shown in Figure 8. The result shows that in year 3 the mine's production plan provides different feasible scheduling options that reflect

the evolution of commodity prices considered. The approach is drastically different from the conventional generation of a single life-of-mine plan, one per different commodity prices considered.

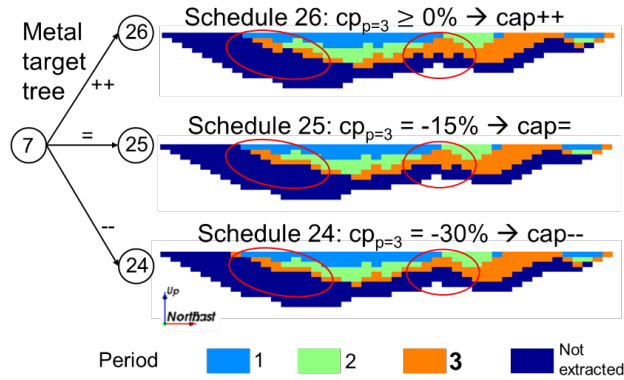


Figure 8: Physical mine production schedules associated to different policies and commodity price evolution.

The results obtained in this case study demonstrate the counter-intuitive joint impact of geological and price uncertainties on decisions policies. For instance, the example shows that an increase of the commodity price does not necessarily imply an increase in metal production and that, similarly, a decrease of the price does not imply a decrease of the metal production. The finite resource and the uncertain supply create a more complex system that cannot be instinctively anticipated. The case study shows that the proposed method allows to consider branching over dynamic price evolution and generate long-term-mine plans that can anticipate the future evolution of commodity prices to take advantage of it and its joint effect with the geological uncertainty.

4 Conclusions

The research experiment presented analyzes the combined influence of both geological and price uncertainties of life-of-mine planning and production scheduling. The proposed approach considers geological uncertainty with an established two-stage stochastic programming model in which extraction and destination decisions are scenario-independent. The commodity price uncertainty is represented in a scenario tree, where branching allows one to take different actions for different price unveilings in a multistage scenario-dependent approach. The proposed method involves pre-computing a pool of mining block subsets that can potentially be extracted together during a given production period and that correspond to a possible metal production quantity target. Then, a stochastic dynamic algorithm defines the best policy, more specifically the metal production target, to choose depending on the evolution of the commodity price. The different approach followed for each source of uncertainty considered is justified by the unavoidable uncertain metal content and pertinent geological properties at any point in time of mining a mineral deposit, while the current commodity price can be considered known at different production times to allow the generation of feasible mine production scheduling options.

The case study presented explored the basics of the proposed approach to identify policies regarding the relations between variations of price and metal production targets that are produced through the combined consideration of both geological and commodity price uncertainties.

Future work should consider larger scale experiments as well as include more constraints to advance practicality and consider a price-dependant definition to the subsets of blocks in the tree of metal production target Γ . Adapting the approach to the more elaborate framework of mining complexes and mineral value chains is a natural extension. The approach will also require a more advanced solving mechanism, such as those based on metaheuristics, in order to handle larger models.

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