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Valuing corporate securities when the firm's assets are illiquid

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Abstract : We use stochastic dynamic programming to design and solve an extended structural setting for which the illiquidity of the firm's assets under liquidation is interpreted as an intangible asset. This corporate security tends to reduce bond values, augment yield spreads, and, thus, partially explain the credit-spread puzzle. To assess our construction, we provide a sensitivity analysis of the values of corporate securities with respect to the illiquidity parameter.

Keywords: Structural model, corporate securities, illiquidity costs, stochastic dynamic programming

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1 Introduction

We revisit the structural model of Ayadi et al. (2016), and extend its associated balance-sheet equality to illiquidity costs. This intangible asset generates a proportional loss on the firm's asset value under liquidation. Defined as such, illiquidity costs tend to decrease bond values, increase yield spreads, and, thus, partially explain the credit-spread puzzle.

The literature on bond values, illiquidity, and the term-structure of interest rates reports a few fundamental properties:

1. There is a positive relationship between illiquidity spreads and yield spreads;
2. This positive relationship is stronger for risky bonds, and holds true regardless of the shape of the term-structure of interest rates.

These properties are discussed from the perspective of econometric models (Chen et al. 2007, Bao et al. 2011, Lin et al. 2011, Friewald et al. 2012, Helwege et al. 2014, and Richardson and Palhares 2019), structural models (Zheng 2006, Ericsson and Renault 2006, He and Xiong 2012, Huang and Huang 2012, He and Milbradt 2014, Abudy and Raviv 2016, and Chen et al. 2018), and reduced-form models (Longstaff et al. 2005, Driessen 2005, Bühler and Trapp 2009, and Berndt 2015).

We use stochastic dynamic programming (SDP) to design and solve an extended structural model that accommodates 1– a large set of Markov state processes, 2– several intangible assets, that is, tax benefits as well as reorganization, illiquidity, and bankruptcy costs, 3– arbitrary coupon and capital payment schedules, 4– multiple seniority classes, and 5– a reorganization process. SDP runs backward in time from the debt maturity to the origin and, at each step of the recursion, alternates between value function approximation and value function integration at earlier evaluation/payment date. Despite the flexibility of the proposed structural setting, its resolution, which combines between SDP and finite elements, is extremely efficient as it assumes only a state-space (but not a time) discretization and a numerical (but not a statistical) error. To assess our construction, we provide a sensitivity analysis of the values of corporate securities with respect to the illiquidity parameter.

The rest of this paper is organized as follows. Section 2 presents the model and its corporate securities. Section 3 is a numerical investigation, while Section 4 concludes.

2 Model and notation

We consider a public company with a debt portfolio whose payment dates belong to $\mathcal{P} = \{t_0 = 0, \ldots, t_N = T^D$. The firm's balance-sheet equality, which holds for all $t \in [0, T]$ and all $a = A_t > 0$, is

$$a + \mathrm{TB}(t, a) - \mathrm{RC}(t, a) - \mathrm{IL}(t, a) - \mathrm{BC}(t, a) = D^s(t, a) + D^j(t, a) + \mathcal{E}(t, a),$$

where $a = A_t$ is the level of the firm's asset value (the state process) at time $t \in [0, T^D]$. The tangible and intangible corporate securities A, TB, RC, IL, BC, D^s, D^j, $D = D^s + D^j$, and $\mathcal{E}$ represent the value of 1– the firm's assets, 2– tax benefits, 2– reorganization costs, 4– illiquidity costs, 5– bankruptcy costs, 6– the senior debt portfolio, 7– the junior debt portfolio, 8– the overall debt portfolio, and 9– the firm's equity, respectively. Extending Leland (1994), we define the total value of the firm TV as the left-hand side of Equation (1). The state process A can be any Markov process consistent with the no-arbitrage principle as long as European vanilla options on A can be valued in closed form.

The firm is committed to paying $d_n^s + d_n^j = d_n$ at $t_n \in \mathcal{P}$ to its creditors, where d_n^s and d_n^j are the outflows generated at t_n by the senior and junior debts, respectively. The total outflow d_n at t_n includes interest payments $C_n = C_n^s + C_n^j$ as well as principal payments $P_n = P_n^s + P_n^j$. Set these payments at zero whenever necessary. The amounts C_n^s, C_n^j, P_n^s, and P_n^j are known to all investors from the very beginning. The last payment dates of the senior and junior debts, both in $\mathcal{P}$, are indicated by T^s and T^j, respectively. Several authors consider a senior coupon bond and a junior coupon bond with a longer maturity, that is, $0 \leq T^s < T^j = T^D$. Senior bondholders are therefore assured capital payment before junior bondholders. This realistic case is embedded in our setting.

For the sake of clarity, we first present the model without a reorganization process, then we describe the full setting. Assume that the evaluation problem has been solved backward in time from the maturity $t_N = T^D$ untill time t_{n+1}, for all levels of the state process. No-arbitrage evaluation and Equation (1) give

$$\begin{aligned} &\mathbb{E}^*_{na}\left[\rho_n A_{t_{n+1}}\right] + \mathbb{E}^*_{na}\left[\rho_n \mathrm{TB}\left(t_{n+1}, A_{t_{n+1}}\right)\right] \\ &- \mathbb{E}^*_{na}\left[\rho_n \mathrm{IL}\left(t_{n+1}, A_{t_{n+1}}\right)\right] - \mathbb{E}^*_{na}\left[\rho_n \mathrm{BC}\left(t_{n+1}, A_{t_{n+1}}\right)\right] \\ =&\mathbb{E}^*_{na}\left[\rho_n D^s\left(t_{n+1}, A_{t_{n+1}}\right)\right] + \mathbb{E}^*_{na}\left[\rho_n D^j\left(t_{n+1}, A_{t_{n+1}}\right)\right] \\ &+ \mathbb{E}^*_{na}\left[\rho_n \mathcal{E}\left(t_{n+1}, A_{t_{n+1}}\right)\right], \end{aligned} \tag{1}$$

where $\rho_n = e^{-r[t_{n+1}-t_n]}$ is the discount factor over $[t_n, t_{n+1}]$, r the risk-free rate (per year), and $\mathbb{E}^*_{na}[.] = \mathbb{E}^*[. \mid A_{t_n} = a]$ the conditional expectation under a risk-neutral probability measure. Any expression of the form $\mathbb{E}^*_{na}\left[\rho_n v\left(t_{n+1}, A_{t_{n+1}}\right)\right]$ is indicated herein by $\overline{v}(t_n, a)$ since it is, indeed, an average value function of t_n and $a = A_{t_n}$, which refers to the present value of a corporate security, based on its future potentialities but not its immediate cash in- or outflow. This legitimizes the interpretation of a corporate security as a financial derivative on the firm's asset value. Alternatively, $\overline{v}(t_n, a)$ can be interpreted as the value function of a corporate security at t_n^+, just after t_n when $A_{t_n^+} = A_{t_n} = a$.

In case of survival at (t_n, a), Equation (1) becomes

$$\begin{aligned} &a + \left[\overline{\mathrm{TB}}(t_n, a) + \mathrm{tb}_n\right] - \overline{\mathrm{IL}}(t_n, a) - \overline{\mathrm{BC}}(t_n, a) \\ &= \left[\overline{D}^s(t_n, a) + d_n^s\right] + \left[\overline{D}^j(t_n, a) + d_n^j\right] + \left[\overline{\mathcal{E}}(t_n, a) - (d_n - \mathrm{tb}_n)\right], \end{aligned} \tag{2}$$

where $\mathrm{tb}_n = C_n \times r^c$ is the tax savings and r^c the periodic corporate tax rate at t_n. The survival condition at (t_n, a), that is,

$$\mathcal{E}(t_n, a) = \overline{\mathcal{E}}(t_n, a) - (d_n - \mathrm{tb}_n) > 0,$$

states that the firm's equity value at t_n^+ exceeds the due payment on the overall debt net of the immediate tax savings at t_n. This condition implicitly assumes that the firm issues new shares of equity at (t_n, a) equivalent to $(d_n - \mathrm{tb}_n) > 0$ (in dollars), as shown by the following equation:

$$\overline{\mathcal{E}}(t_n, a) = \mathcal{E}(t_n, a) + (d_n - \mathrm{tb}_n) > \mathcal{E}(t_n, a).$$

This rule can be seen as a protection covenant for the firm's bondholders. Since the firm's equity value at time t_n is a continuous and increasing function of the state variable, there exists a default (liquidation) barrier b_n under which the firm is liquidated, that is,

$$\mathcal{E}(t_n, a) = 0, \quad \text{for } a \leq b_n,$$

while the strict priority rule applies, which highlights the compound-call-option nature of the residual asset

$$\mathcal{E}(t_n, a) = \max\left[0, \overline{\mathcal{E}}(t_n, a) - (d_n - \mathrm{tb}_n)\right],$$

as claimed by Geske (1977).

Under liquidation, we observe a first loss on the firm's asset value at the rate v, due to illiquidity costs, followed by a second loss at the rate w, due to bankruptcy costs. The remaining firm's asset value $\overline{wv}a = (1-w)(1-v)\,a$ is used to (partially) pay bondholders by their seniority class. It can happen that, under liquidation at time t_n, senior bondholders are paid partially, while junior bondholders are not paid at all, in which case, there exists a threshold $0 < b_n^s < b_n$ such that

$$D^j(t_n, a) = 0, \quad \text{for } a \leq b_n^s.$$

The barrier b_n^s can be interpreted as a loss barrier for senior bondholders. This version of the model includes the seminal works of Merton (1974), Black and Cox (1976), Geske (1977), and Leland (1994).

The gain in flexibility comes with a minor loss of efficiency. Their explicit approach is exchanged here for a quasi-explicit approach based on SDP and finite elements.

Table 1 exhibits the value functions of corporate securities at t_n, for $n = 0, \ldots, N-1$, while Table 2 points out their limit conditions at $t_N = T^D$. At maturity, the default barrier is $b_N = (d_N - \mathrm{tb}_N)$ and the loss barrier for senior bondholders is $b_N^s = \max\left(\overline{wv}a - d_N^s, 0\right)$.

Table 1: Value functions before maturity without reorganization

BSE	Liquidation $a \le b_n$	Survival $a > b_n$
$+a = A_{t_n}$	a	a
$+\mathrm{TB}(t_n, a)$	0	$\overline{\mathrm{TB}}\,(t_n, a) + \mathrm{tb}_n$
$-\mathrm{IL}(t_n, a)$	$-va$	$-\overline{\mathrm{IL}}\,(t_n, a)$
$-\mathrm{BC}(t_n, a)$	$-w\overline{v}a$	$-\overline{\mathrm{BC}}\,(t_n, a)$
$=$	$=$	$=$
$+D^s\,(t_n, a)$	$\min\left[\overline{wv}a, \overline{D}^s\,(t_n, a) + d_n^s\right]$	$\overline{D}^s\,(t_n, a) + d_n^s$
$+D^j\,(t_n, a)$	$\max\left[0, \overline{wv}a - D^s\,(t_n, a)\right]$	$\overline{D}^j\,(t_n, a) + d_n^j$
$+\mathcal{E}\,(t_n, a)$	0	$\overline{\mathcal{E}}\,(t_n, a) - (d_n - \mathrm{tb}_n)$

We now introduce a reorganization process for which bondholders assume a moderate immediate loss under default in exchange for a substantial future gain. We propose a design where bondholders consent to reduce their promized cash-inflow payment at t_n at a grace rate $\eta \in [0,1]$, as long as the firm is under default and the number of grace periods asked for by the firm before t_n doesn't exceed $\overline{g} \in \mathbb{N}$. The variables η and $\overline{g}$ can be seen as the design parameters of the proposed reorganization process. For example, let $\eta = 10\%$, $\overline{g} = 3$, and $g_n = 2$, while the firm is under default at (t_n, a). Thus, the firm is better off asking for a third and last reorganization period, which reduces its due payment d_n by $\eta = 10\%$ and expands its non-bankrupt event. Broadie and Kaya (2007) show that reorganization processes based on partial forgiveness can augment the total value of the firm, and that the benefits drastically decrease with $\overline{g}$. In sum, short-time reorganization processes are recommended. Our numerical investigation confirms their findings. Tables 3–4 exhibit the SDP value functions in the full setting with a reorganization process.

Table 2: Value functions at maturity without reorganization

BSE	Liquidation $a \le b_N$	Survival $a > b_N$
$+a = A_{t_N}$	a	a
$+\mathrm{TB}(t_N, a)$	0	tb_N
$-\mathrm{IL}(t_N, a)$	$-va$	0
$-\mathrm{BC}(t_N, a)$	$-w\overline{v}a$	0
$=$	$=$	$=$
$+D^s\,(t_N, a)$	$\min\left(\overline{wv}a, d_N^s\right)$	d_N^s
$+D^j\,(t_N, a)$	$\max\left(0, \overline{wv}a - d_N^s\right)$	d_N^j
$+\mathcal{E}\,(t_N, a)$	0	$a - (d_N - \mathrm{tb}_N)$

Table 3: Value functions before maturity with reorganization

BSE	Reorganization $a \in \left[b_n^l\,(g), b_n\,(g)\right]$	Survival $a > b_n\,(g)$
$+a = A_{t_n}$	a	a
$+\mathrm{TB}(t_n, a, g)$	$\overline{\mathrm{TB}}\,(t_n, a, g+1) + \overline{\eta}{\times}\mathrm{tb}_n$	$\overline{\mathrm{TB}}\,(t_n, a, g) + \mathrm{tb}_n$
$-\mathrm{RC}(t_n, a, g)$	$-\overline{\mathrm{RC}}\,(t_n, a, g+1) - ua$	$-\overline{\mathrm{RC}}\,(t_n, a, g)$
$-\mathrm{IL}(t_n, a, g)$	$-\overline{\mathrm{IL}}\,(t_n, a, g+1)$	$-\overline{\mathrm{IL}}\,(t_n, a, g)$
$-\mathrm{BC}(t_n, a, g)$	$-\overline{\mathrm{BC}}\,(t_n, a, g+1)$	$-\overline{\mathrm{BC}}\,(t_n, a, g)$
$=$	$=$	$=$
$+D^s\,(t_n, a, g)$	$\overline{D}^s\,(t_n, a, g+1) + \overline{\eta}d_n^s$	$\overline{D}^s\,(t_n, a, g) + d_n^s$
$+D^j\,(t_n, a, g)$	$\overline{D}^j\,(t_n, a, g+1) + \overline{\eta}d_n^j$	$\overline{D}^j\,(t_n, a, g) + d_n^j$
$+\mathcal{E}\,(t_n, a, g)$	$\overline{\mathcal{E}}\,(t_n, a, g+1) - \left[\overline{\eta}\,(d_n - \mathrm{tb}_n) + ua\right]$	$\overline{\mathcal{E}}\,(t_n, a, g) - (d_n - \mathrm{tb}_n)$

Table 3: (continued):Value functions before maturity with reorganization

BSE	Liquidation: $a < b_n^l(g)$
$+a = A_{t_n}$	a
$+\mathrm{TB}(t_n, a, g)$	0
$-\mathrm{RC}(t_n, a, g)$	0
$-\mathrm{IL}(t_n, a, g)$	$-va$
$-\mathrm{BC}(t_n, a, g)$	$-w\overline{v}a$
$=$	$=$
$+D^s(t_n, a, g)$	$\min\left[\overline{wv}a, \overline{D}^s(t_n, a, g) + d_n^s\right]$
$+D^j(t_n, a, g)$	$\max\left[0, \overline{wv}a - D^s(t_n, a, g)\right]$
$+\mathcal{E}(t_n, a, g)$	0

The value function of a corporate security becomes a function of time, the level of the firm's asset value, and the number of reorganizations requested by the firm before that time. For $g \in \{0, \ldots, \overline{g}\}$ and $\eta \in [0, 1]$, the balance-sheet equality in Equation (2) is exchanged for

$$\begin{aligned} & a + \left[\overline{\mathrm{TB}}(t_n, a, g) + \mathrm{tb}_n\right] - \overline{\mathrm{RC}}(t_n, a, g) \\ & - \overline{\mathrm{IL}}(t_n, a, g) - \overline{\mathrm{BC}}(t_n, a, g) \\ = & \left[\overline{D}^s(t_n, a, g) + d_n^s\right] + \left[\overline{D}^j(t_n, a, g) + d_n^j\right] \\ & + \left[\overline{\mathcal{E}}(t_n, a, g) - (d_n - \mathrm{tb}_n)\right], \end{aligned}$$

under survival and

$$\begin{aligned} & a + \left[\overline{\mathrm{TB}}(t_n, a, g+1) + \overline{\eta}\mathrm{tb}_n\right] - \left[\overline{\mathrm{RC}}(t_n, a, g+1) + ua\right] \\ & - \overline{\mathrm{IL}}(t_n, a, g+1) - \overline{\mathrm{BC}}(t_n, a, g+1) \\ = & \left[\overline{D}^s(t_n, a, g+1) + \overline{\eta} d_n^s\right] + \left[\overline{D}^j(t_n, a, g+1) + \overline{\eta} d_n^j\right] \\ & + \left[\overline{\mathcal{E}}(t_n, a, g+1) - (\overline{\eta}(d_n - \mathrm{tb}_n) + ua)\right], \end{aligned}$$

under reorganization, where $u \in [0, 1]$ is the reorganization parameter and $\overline{u} = 1 - u$. This results in a survival event characterized by

$$\overline{\mathcal{E}}(t_n, a, g) - (d_n - \mathrm{tb}_n) > 0, \quad \text{while } g \leq \overline{g},$$

and a reorganization event characterized by

$$\overline{\mathcal{E}}(t_n, a, g) - (d_n - \mathrm{tb}_n) \leq 0$$

and

$$\overline{\mathcal{E}}(t_n, a, g+1) - [\overline{\eta}(d_n - \mathrm{tb}_n) + ua] > 0,$$

while $g < \overline{g}$.

Table 4: Value functions at maturity with reorganization

BSE	Liquidation	Reorganization	Survival
$+a = A_{t_N}$	a	a	a
$+\mathrm{TB}(t_N, a, g)$	0	$\overline{\eta}\mathrm{tb}_N$	tb_N
$-\mathrm{RC}(t_N, a, g)$	0	$-ua$	0
$-\mathrm{IL}(t_N, a, g)$	$-va$	0	0
$-\mathrm{BC}(t_N, a, g)$	$-w\overline{v}a$	0	0
$=$	$=$	$=$	$=$
$+D^s(t_N, a, g)$	$\min\left[\overline{wv}a, d_N^s\right]$	$\overline{\eta} d_N^s$	d_N^s
$+D^j(t_N, a, g)$	$\max\left[0, \overline{wv}a - D^s(t_N, a, g)\right]$	$\overline{\eta} d_N^j$	d_N^j
$+\mathcal{E}(t_N, a, g)$	0	$\overline{u}a - \overline{\eta}(d_N - \mathrm{tb}_N)$	$a - (d_N - \mathrm{tb}_N)$

For example, $\mathrm{TB}(t_n, a, g) = \overline{\mathrm{TB}}(t_n, a, g+1) + \overline{\eta} \times \mathrm{tb}_n$ in Table 3 indicates that the firm asks for a grace period at t_n since the firm has asked for g grace periods before t_n and $g + 1$ before t_n^+. A reorganization can take place over $[t_n, t_{n+1}]$ and be effective if $\overline{\eta}$ and u are low, which suggests a liquidation barrier $b_n^l(g)$ below the default (reorganization) barrier $b_n(g)$. In sum, at each step of the recursion, SDP computes $\overline{g}+1$ value functions per corporate security as well as $\overline{g}+1$ default (reorganization) and liquidation barriers. Since the firm's equity value cannot be deteriorated with a reorganization process based on partial forgiveness, we define the optimal reorganization design as the solution of the following optimization problem:

$$\begin{array}{l}\sup D\left(t_0, A_{t_0}, g_0\right) \\ \overline{g} < N \text{ and } \eta \in [0,1]\end{array},$$

where $g_0 \leq \overline{g}$ is the number of grace periods called for by the firm before t_0 with the convention that $g_0 = 0$ when the count starts at t_0. This optimization problem is solved on a grid mesh of $(\overline{g}, \eta)$. The proposed reorganization process is case sensitive in the sense that the number of allowed grace periods and the grace rate depend on the firm's characteristics.

3 Numerical investigation

The debt portfolio is reduced herein to a coupon bond. We perform a sensitivity analysis of SDP value functions with respect to the bond's coupon rate then to the bond's time to maturity. For each experiment, we vary the illiquidity parameter v in {0%, 20%, 40%}.

It is worth noticing that the firm's bond value D is sensitive to the illiquidity parameter v, but the firm's equity value $\mathcal{E}$ is not as it's null under liquidation, where illiquidity costs are effective. Consider two identical companies except for their illiquidity parameters $v < v'$. Since the illiquidity assumption on the firm's assets propagate naturally to the firm's debt portfolio, one has $D(v) > D(v')$ or, equivalently,

$$\mathrm{LS} = \mathrm{YS}(v') - \mathrm{YS}(v) > 0,$$

where YS stands for the yield spread and LS for the liquidity spread associated to v and v'. By construction, the higher is $v \in [0,1]$ (ceteris paribus) and the larger is the loss of value for bondholders under liquidation.

A higher coupon rate results in two conflicting effects on the firm's debt value: 1– a higher promised cash-flow stream, which has a positive impact on the debt value, and 2– a higher default probability, which has a negative impact on the debt value. The opposite holds true for the firm's equity value, seen as a residual claim in the left-hand side of the balance-sheet equality (1). Figure 1 shows that the firm's equity value is a decreasing function of the coupon rate. The default-probability effect dominates. The same value function remains insensitive to the illiquidity parameter. Figure 2 locates the firm's maximum debt capacity when the coupon rate changes. The cash-flow effect dominates for investment-grade bonds and the default-probability effect dominates for speculative-grade bonds. In all situations, a higher illiquidity parameter exhibits a lower firm's bond value. Figure 3 clearly shows a positive relationship between illiquidity spreads and yield spreads, which becomes stronger for risky bonds, where illiquidity adverse effects are accentuated for high-yield bonds. Thus, the illiquidity assumption on the firm's assets under liquidation partially explains the credit-spread puzzle. The total value of the firm in Figure 4 is sensitive to the debt–equity ratio and shows a global maximum when the coupon rate changes. Indeed, illiquidity costs tend to lower the total value of the firm. Tax benefits in Figure 5 are represented by an increasing (a decreasing) function of the coupon rate for high- (low-) quality bonds. For low coupon rates, the cash-flow effect dominates, while, for high-coupon rates, the default-probability effect dominates. Tax benefits are insensitive to the illiquidity parameter as the former is null under liquidation. While Figure 6 reports illiquidity costs as an increasing function of the coupon rate and of the illiquidity parameter, Figure 7 reports bankruptcy costs as an increasing function of the coupon rate and a decreasing function of the illiquidity parameter. The firm's total default probability in Figure 8 is an increasing function of the coupon rate, but remains insensitive to the illiquidity parameter as illiquidity costs have no effect on default barriers.

Our experiments highlight multiple forms of the term-structure of yield spreads and of liquidity spreads, depending on the firm's credit quality. In all cases, the positive relationship between illiquidity spreads and yield spreads holds true. Figure 9 and Figure 10 exhibit a decreasing term-structure of yield spreads and of liquidity spreads, which characterize risky environments. A parallel can be made with the literature on riskless bonds (Amihud and Mendelson 1991, Longstaff 2004, and Kempf et al. 2012).

The proposed reorganization process is still valuable in the presence of illiquidity costs. Figures 11–13 point out that a higher illiquidity parameter results in a different reorganization plan with a larger grace rate and number of grace periods. For $v = 0\%$, 20%, and 40%, the optimal reorganization plans are $(\eta = 40\%, \overline{g} = 1)$, $(\eta = 60\%, \overline{g} = 1)$, and $(\eta = 70\%, \overline{g} = 3)$, respectively.

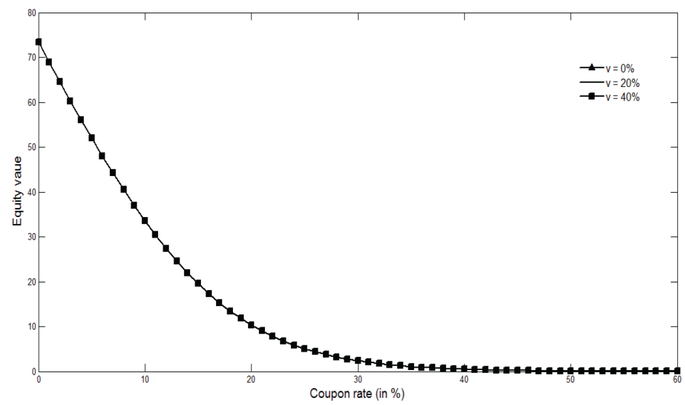

Figure 1: Equity value as a function of the bond's coupon. The debt is a coupon bond with a maturity of 10 years and a principal amount of \$100. Set $A_0 = \$120, \sigma = 30\%, r = 6\%, r^c = 35\%, \eta = 0, v \in \{0\%, 20\%, 40\%\}$, and $\omega = 25\%$.

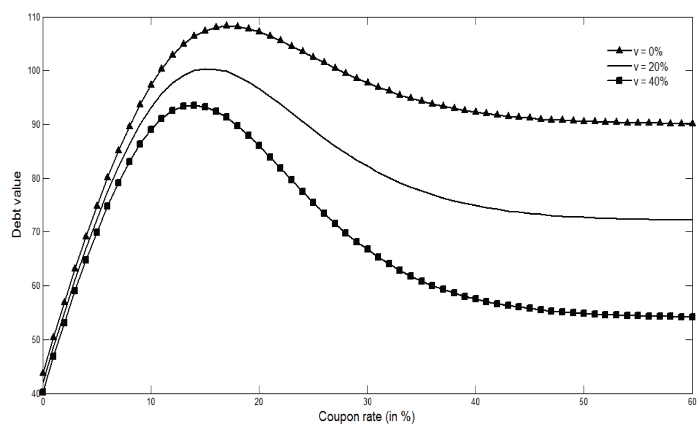

Figure 2: Debt value as a function of the bond's coupon. The debt is a coupon bond with a maturity of 10 years and a principal amount of \$100. Set $A_0 = \$120, \sigma = 30\%, r = 6\%, r^c = 35\%, \eta = 0, v \in \{0\%, 20\%, 40\%\}$, and $\omega = 25\%$.

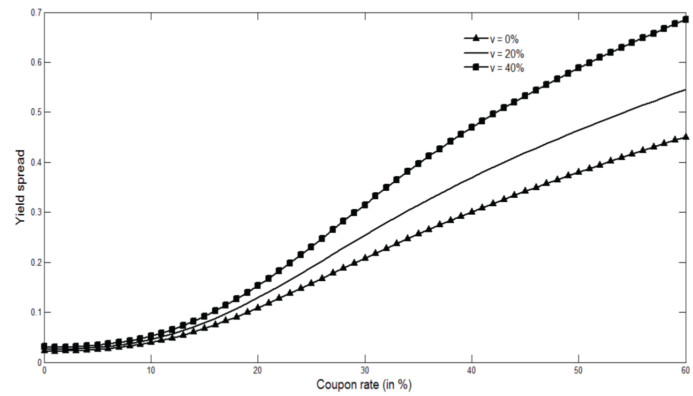

Figure 3: Yield spread as a function of the bond's coupon. The debt is a coupon bond with a maturity of 10 years and a principal amount of \$100. Set $A_0 = \$120, \sigma = 30\%, r = 6\%, r^c = 35\%, \eta = 0, \upsilon \in \{0\%, 20\%, 40\%\},$ and $\omega = 25\%$.

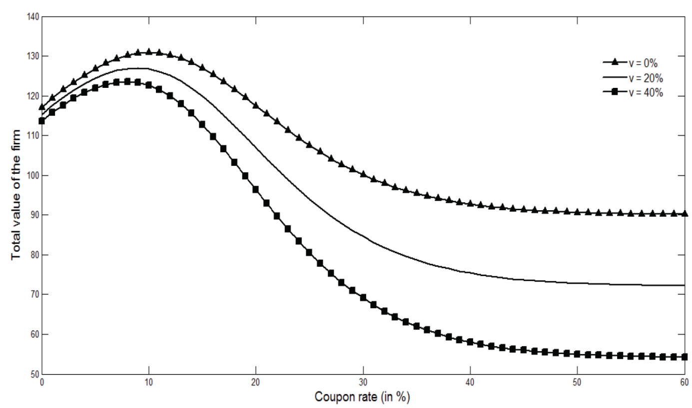

Figure 4: Total value of the firm as a function of the bond's coupon. The debt is a coupon bond with a maturity of 10 years and a principal amount of \$100. Set $A_0 = \$120, \sigma = 30\%, r = 6\%, r^c = 35\%, \eta = 0, \upsilon \in \{0\%, 20\%, 40\%\},$ and $\omega = 25\%$.

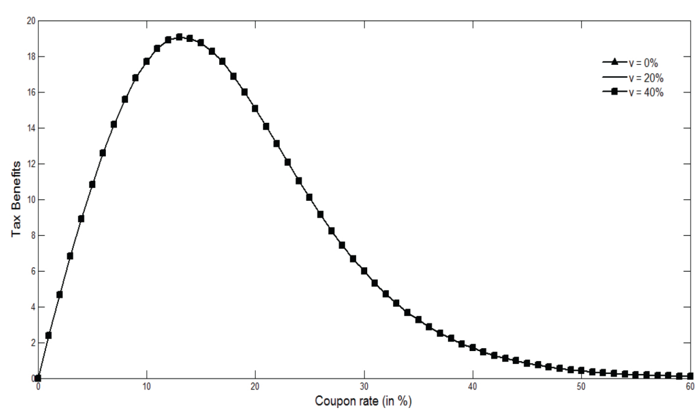

Figure 5: Tax benefits as a function of the bond's coupon. The debt is a coupon bond with a maturity of 10 years and a principal amount of \$100. Set $A_0 = \$120, \sigma = 30\%, r = 6\%, r^c = 35\%, \eta = 0, \upsilon \in \{0\%, 20\%, 40\%\},$ and $\omega = 25\%$.

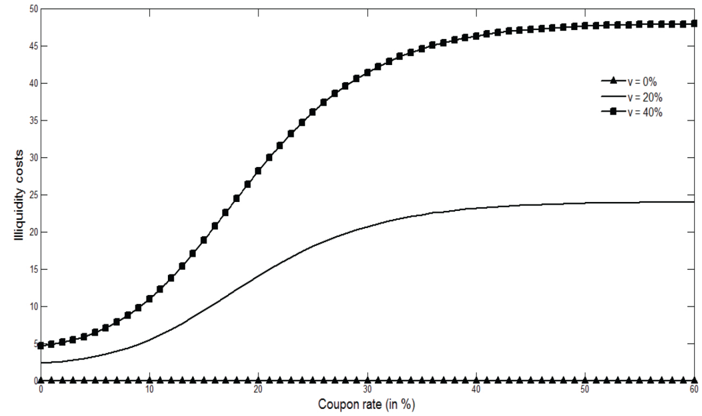

Figure 6: Illiquidity costs as a function of the bond's coupon. The debt is a coupon bond with a maturity of 10 years and a principal amount of \$100. Set $A_0 = \$120, \sigma = 30\%, r = 6\%, r^c = 35\%, \eta = 0, \upsilon \in \{0\%, 20\%, 40\%\},$ and $\omega = 25\%$.

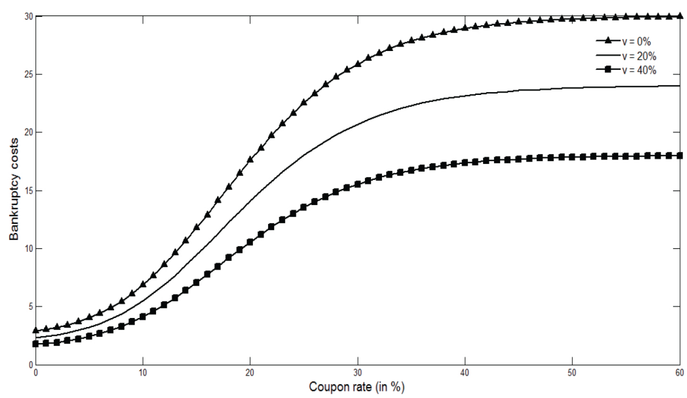

Figure 7: Bankruptcy costs as a function of the bond's coupon. The debt is a coupon bond with a maturity of 10 years and a principal amount of \$100. Set $A_0 = \$120, \sigma = 30\%, r = 6\%, r^c = 35\%, \eta = 0, \upsilon \in \{0\%, 20\%, 40\%\},$ and $\omega = 25\%$.

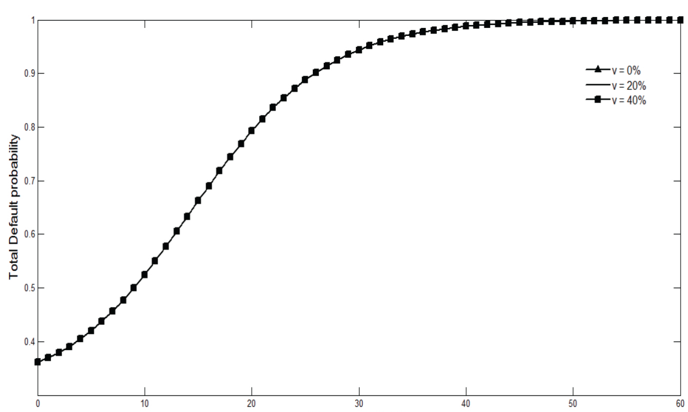

Figure 8: Total default probability as a function of the bond's coupon. The debt is a coupon bond with a maturity of 10 years and a principal amount of \$100. Set $A_0 = \$120, \sigma = 30\%, r = 6\%, r^c = 35\%, \eta = 0, \upsilon \in \{0\%, 20\%, 40\%\},$ and $\omega = 25\%$.

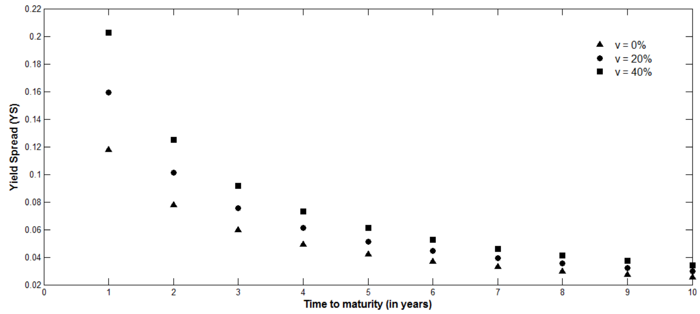

Figure 9: Term structure of yield spreads. The debt is acoupon bond with a maturity of 10 years, a principal amount of \$100; and an annual coupon rate of 5%. Set $A_0 = \$200, \sigma = 30\%, r = 6\%, r^c = 35\%, \eta = 0, \upsilon \in \{0\%, 20\%, 40\%\},$ and $\omega = 25\%$.

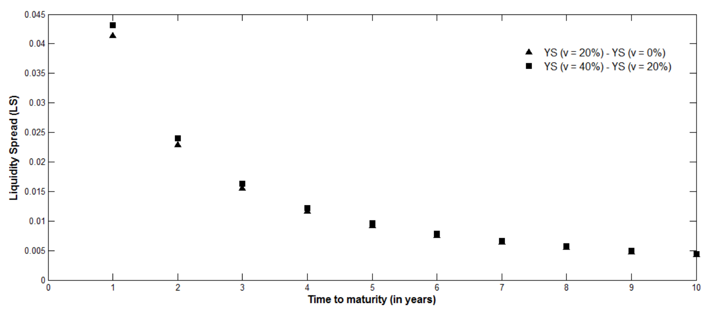

Figure 10: Term structure of liquidity spreads. The debt is a coupon bond with a maturity of 10 years, a principal amount of \$100; and an annual coupon rate of 5%. Set $A_0 = \$200, \sigma = 30\%, r = 6\%, r^c = 35\%, \eta = 0, \upsilon \in \{0\%, 20\%, 40\%\},$ and $\omega = 25\%$.

$\overline{g}$ \ η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
10	(74.78, 52.02)	(74.94, 52.04)	(75.21, 52.20)	(75.41, 52.55)	(75.52, 53.10)	(75.45, 53.90)	(75.13, 54.98)	(74.52, 56.37)	(73.43, 58.07)	(71.74, 60.02)	(69.68, 62.11)
9	(74.78, 52.02)	(74.94, 52.04)	(75.21, 52.20)	(75.41, 52.55)	(75.52, 53.10)	(75.45, 53.90)	(75.13, 54.98)	(74.52, 56.37)	(73.43, 58.07)	(71.74, 60.02)	(69.68, 62.11)
8	(74.78, 52.02)	(74.94, 52.04)	(75.21, 52.20)	(75.41, 52.55)	(75.52, 53.10)	(75.45, 53.90)	(75.13, 54.98)	(74.52, 56.37)	(73.43, 58.07)	(71.74, 60.02)	(69.68, 62.11)
7	(74.78, 52.02)	(74.94, 52.04)	(75.21, 52.20)	(75.41, 52.55)	(75.52, 53.10)	(75.45, 53.90)	(75.13, 54.98)	(74.52, 56.37)	(73.43, 58.07)	(71.74, 60.02)	(69.68, 62.11)
6	(74.78, 52.02)	(74.94, 52.04)	(75.21, 52.20)	(75.41, 52.55)	(75.52, 53.10)	(75.45, 53.90)	(75.13, 54.98)	(74.52, 56.37)	(73.43, 58.07)	(71.74, 60.02)	(69.68, 62.11)
5	(74.78, 52.02)	(74.94, 52.04)	(75.21, 52.20)	(75.41, 52.55)	(75.52, 53.10)	(75.45, 53.90)	(75.13, 54.98)	(74.52, 56.37)	(73.43, 58.07)	(71.74, 60.02)	(69.68, 62.11)
4	(74.78, 52.02)	(74.94, 52.04)	(75.21, 52.20)	(75.41, 52.55)	(75.52, 53.10)	(75.45, 53.90)	(75.13, 54.98)	(74.52, 56.37)	(73.43, 58.07)	(71.74, 60.02)	(69.68, 62.11)
3	(74.78, 52.02)	(74.94, 52.04)	(75.21, 52.20)	(75.41, 52.55)	(75.52, 53.10)	(75.45, 53.90)	(75.13, 54.98)	(74.52, 56.37)	(73.42, 58.07)	(71.73, 60.02)	(69.68, 62.11)
2	(74.78, 52.02)	(74.94, 52.04)	(75.21, 52.20)	(75.41, 52.55)	(75.52, 53.10)	(75.45, 53.90)	(75.13, 54.98)	(74.51, 56.37)	(73.41, 58.07)	(71.72, 60.02)	(69.68, 62.11)
1	(74.78, 52.02)	(74.94, 52.04)	(75.21, 52.20)	(75.41, 52.55)	**(75.52, 53.10)**	(75.45, 53.90)	(75.13, 54.98)	(74.46, 56.36)	(73.32, 58.06)	(71.68, 60.01)	(69.67, 62.10)
0	(74.78, 52.02)	(74.78, 52.02)	(74.78, 52.02)	(74.78, 52.02)	(74.78, 52.02)	(74.78, 52.02)	(74.78, 52.02)	(74.78, 52.02)	(74.78, 52.02)	(74.78, 52.02)	(74.78, 52.02)

Figure 11: Debt and equity as functions of η and $\overline{g}$. The debt is a 5% coupon bond with a maturity of 10 years and a principal amount of \$100. Set $A_0 = \$120, \sigma = 30\%, r = 6\%, r^c = 35\%, u = 5\%, \upsilon = 0\%$ and $\omega = 25\%$.

$\overline{g}$ \ η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
10	(72.37, 52.02)	(72.65, 52.04)	(73.19, 52.20)	(73.69, 52.55)	(74.14, 53.10)	(74.43, 53.90)	(74.45, 54.98)	(74.18, 56.37)	(73.34, 58.07)	(71.73, 60.02)	(69.68, 62.11)
9	(72.37, 52.02)	(72.65, 52.04)	(73.19, 52.20)	(73.69, 52.55)	(74.14, 53.10)	(74.43, 53.90)	(74.45, 54.98)	(74.18, 56.37)	(73.34, 58.07)	(71.73, 60.02)	(69.68, 62.11)
8	(72.37, 52.02)	(72.65, 52.04)	(73.19, 52.20)	(73.69, 52.55)	(74.14, 53.10)	(74.43, 53.90)	(74.45, 54.98)	(74.18, 56.37)	(73.34, 58.07)	(71.73, 60.02)	(69.68, 62.11)
7	(72.37, 52.02)	(72.65, 52.04)	(73.19, 52.20)	(73.69, 52.55)	(74.14, 53.10)	(74.43, 53.90)	(74.45, 54.98)	(74.18, 56.37)	(73.34, 58.07)	(71.73, 60.02)	(69.68, 62.11)
6	(72.37, 52.02)	(72.65, 52.04)	(73.19, 52.20)	(73.69, 52.55)	(74.14, 53.10)	(74.43, 53.90)	(74.45, 54.98)	(74.18, 56.37)	(73.34, 58.07)	(71.73, 60.02)	(69.68, 62.11)
5	(72.37, 52.02)	(72.65, 52.04)	(73.19, 52.20)	(73.69, 52.55)	(74.14, 53.10)	(74.43, 53.90)	(74.45, 54.98)	(74.18, 56.37)	(73.34, 58.07)	(71.73, 60.02)	(69.68, 62.11)
4	(72.37, 52.02)	(72.65, 52.04)	(73.19, 52.20)	(73.69, 52.55)	(74.14, 53.10)	(74.43, 53.90)	(74.45, 54.98)	(74.18, 56.37)	(73.33, 58.07)	(71.73, 60.02)	(69.68, 62.11)
3	(72.37, 52.02)	(72.65, 52.04)	(73.19, 52.20)	(73.69, 52.55)	(74.14, 53.10)	(74.43, 53.90)	(74.45, 54.98)	(74.18, 56.37)	(73.33, 58.07)	(71.72, 60.02)	(69.68, 62.11)
2	(72.37, 52.02)	(72.65, 52.04)	(73.19, 52.20)	(73.69, 52.55)	(74.14, 53.10)	(74.43, 53.90)	(74.45, 54.98)	(74.17, 56.37)	(73.30, 58.07)	(71.71, 60.02)	(69.68, 62.11)
1	(72.37, 52.02)	(72.65, 52.04)	(73.19, 52.20)	(73.69, 52.55)	(74.14, 53.10)	(74.42, 53.90)	**(74.45, 54.98)**	(74.08, 56.36)	(73.17, 58.06)	(71.65, 60.01)	(69.67, 62.10)
0	(72.37, 52.02)	(72.37, 52.02)	(72.37, 52.02)	(72.37, 52.02)	(72.37, 52.02)	(72.37, 52.02)	(72.37, 52.02)	(72.37, 52.02)	(72.37, 52.02)	(72.37, 52.02)	(72.37, 52.02)

Figure 12: Debt and equity as functions of η and $\overline{g}$. The debt is a 5% coupon bond with a maturity of 10 years and a principal amount of \$100. Set $A_0 = \$120, \sigma = 30\%, r = 6\%, r^c = 35\%, u = 5\%, \upsilon = 20\%$ and $\omega = 25\%$.

$\overline{g}$ \ η	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
10	(69.96, 52.02)	(70.36, 52.04)	(71.17, 52.20)	(71.97, 52.55)	(72.76, 53.10)	(73.40, 53.90)	(73.78, 54.98)	(73.85, 56.37)	(73.24, 58.07)	(71.72, 60.02)	(69.68, 62.11)
9	(69.96, 52.02)	(70.36, 52.04)	(71.17, 52.20)	(71.97, 52.55)	(72.76, 53.10)	(73.40, 53.90)	(73.78, 54.98)	(73.85, 56.37)	(73.24, 58.07)	(71.72, 60.02)	(69.68, 62.11)
8	(69.96, 52.02)	(70.36, 52.04)	(71.17, 52.20)	(71.97, 52.55)	(72.76, 53.10)	(73.40, 53.90)	(73.78, 54.98)	(73.85, 56.37)	(73.24, 58.07)	(71.72, 60.02)	(69.68, 62.11)
7	(69.96, 52.02)	(70.36, 52.04)	(71.17, 52.20)	(71.97, 52.55)	(72.76, 53.10)	(73.40, 53.90)	(73.78, 54.98)	(73.85, 56.37)	(73.24, 58.07)	(71.72, 60.02)	(69.68, 62.11)
6	(69.96, 52.02)	(70.36, 52.04)	(71.17, 52.20)	(71.97, 52.55)	(72.76, 53.10)	(73.40, 53.90)	(73.78, 54.98)	(73.85, 56.37)	(73.24, 58.07)	(71.72, 60.02)	(69.68, 62.11)
5	(69.96, 52.02)	(70.36, 52.04)	(71.17, 52.20)	(71.97, 52.55)	(72.76, 53.10)	(73.40, 53.90)	(73.78, 54.98)	(73.85, 56.37)	(73.24, 58.07)	(71.72, 60.02)	(69.68, 62.11)
4	(69.96, 52.02)	(70.36, 52.04)	(71.17, 52.20)	(71.97, 52.55)	(72.76, 53.10)	(73.40, 53.90)	(73.78, 54.98)	(73.85, 56.37)	(73.24, 58.07)	(71.72, 60.02)	(69.68, 62.11)
3	(69.96, 52.02)	(70.36, 52.04)	(71.17, 52.20)	(71.97, 52.55)	(72.76, 53.10)	(73.40, 53.90)	(73.78, 54.98)	**(73.85, 56.37)**	(73.22, 58.07)	(71.71, 60.02)	(69.68, 62.11)
2	(69.96, 52.02)	(70.36, 52.04)	(71.17, 52.20)	(71.97, 52.55)	(72.76, 53.10)	(73.40, 53.90)	(73.78, 54.98)	(73.83, 56.37)	(73.19, 58.07)	(71.69, 60.02)	(69.68, 62.11)
1	(69.96, 52.02)	(70.36, 52.04)	(71.17, 52.20)	(71.97, 52.55)	(72.76, 53.10)	(73.40, 53.90)	(73.77, 54.98)	(73.71, 56.36)	(73.02, 58.06)	(71.61, 60.01)	(69.67, 62.10)
0	(69.96, 52.02)	(69.96, 52.02)	(69.96, 52.02)	(69.96, 52.02)	(69.96, 52.02)	(69.96, 52.02)	(69.96, 52.02)	(69.96, 52.02)	(69.96, 52.02)	(69.96, 52.02)	(69.96, 52.02)

Figure 13: Debt and equity as functions of η and $\overline{g}$. The debt is a 5% coupon bond with a maturity of 10 years and a principal amount of \$100. Set $A_0 = \$120, \sigma = 30\%, r = 6\%, r^c = 35\%, u = 5\%, \upsilon = 40\%$ and $\omega = 25\%$.

4 Conclusion

We propose an extended structural model that accommodates 1– a large set of Markov state processes, 2– several intangible assets, that is, tax benefits as well as reorganization, illiquidity, and bankruptcy costs, 3– arbitrary coupon and capital payment schedules, 4– multiple seniority classes, and 5– a reorganization process. The model resolution is achieved via stochastic dynamic programming. Our numerical investigation is in line with the literature on bond values, illiquidity, and the term-structure of interest rates.

A future research avenue consists of estimating the model parameters and performing an empirical investigation on public companies. The fact that the firm's equity value is insensitive to the illiquidity parameter is a challenging issue to solve.

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